1. Assuming zero body forces, investigate the candidacy of the following functions as Airy Stress function:

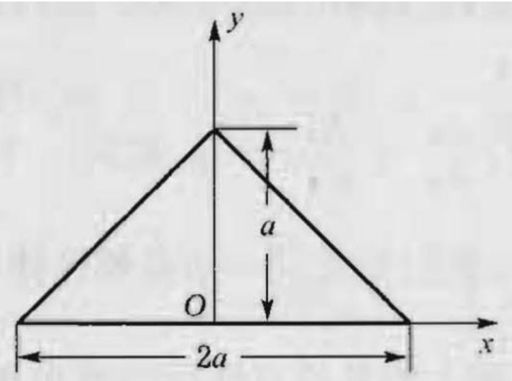
$$(1)\ \psi = ax^2y; \quad (2)\ \psi = axy^3; \quad (3)\ \psi = ay^4; \quad (4)\ \psi = ay^5,$$

where a is a constant. If yes, calculate the resultant stress components. For a rectangular domain, label the corresponding stress tractions by using boundary conditions.

2. Assuming zero body forces, calculate and label both the normal and shear stresses on the three sides of the following triangular domain due to the Airy Stress Function:

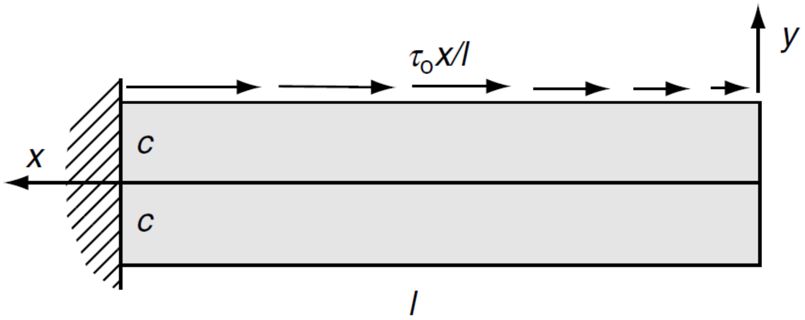
$$\psi = a\left(x^3 + xy^2\right),$$

where a is a constant.



3. Assuming zero body forces, determine the boundary conditions for the following cantilever beam subjected to a linearly distributed shear traction $\tau = \tau_0 x/l$. Also, determine the eight constants for the given Airy Stress Function:

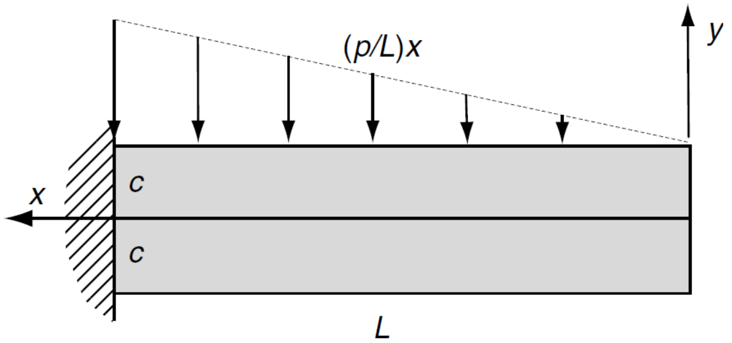
$$\psi = C_1 y^2 + C_2 y^3 + C_3 y^4 + C_4 y^5 + C_5 x^2 + C_6 x^2 y + C_7 x^2 y^2 + C_8 x^2 y^3 .$$



4. The following Airy Stress Function

$$\psi = C_1 xy + C_2 \frac{x^3}{6} + C_3 \frac{x^3 y}{6} + C_4 \frac{xy^3}{6} + C_5 \frac{x^3 y^3}{9} + C_6 \frac{xy^5}{20},$$

is proposed to solve the problem of a cantilever beam carrying at uniformly varying loading as shown in the following figure. Explicitly verify that this stress function will satisfy all conditions on the problem and determine each of the constants C_i and resulting stress field. Use resultant force boundary conditions at the beam-ends.



5. (Optional) A triangular plate of narrow rectangular cross-section and uniform thickness is loaded uniformly along its top edge as shown in the figure. Verify that the Airy Stress Function

$$\psi = \frac{p\cot\alpha}{2(1-\alpha\cot\alpha)}\left[-x^2\tan\alpha + xy + \left(x^2+y^2\right)\left(\alpha - \tan^{-1}\frac{y}{x}\right)\right],$$

solves this plane problem. For the particular case of $\alpha = 30^\circ$, explicitly calculate the normal and shear stress distribution over a typical cross-section *AB* and make comparison plots (MATLAB recommended) of your results with those from elementary strength of materials.