1. For a beam of circular cross-section, analysis from elementary strength of materials theory yields the following stresses:

$$\sigma_x = -\frac{My}{I}, \quad \tau_{xy} = \frac{V\left(R^2 - y^2\right)}{3I}, \quad \sigma_y = \sigma_z = \tau_{xz} = \tau_{yz} = 0$$

where R is the section radius, I is the moment of inertia, M is the bending moment, and V is the shear force. Assuming zero body forces, show that these stresses do not satisfy the equilibrium equations. This result is one of many that indicate the approximate nature of strength of materials theory.

2. A *hydrostatic* stress field is specified by

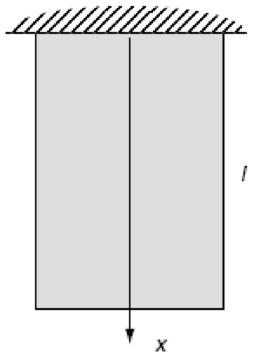
$$\sigma_{ij} = -p\delta_{ij} = \begin{bmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{bmatrix}$$

where $p = p(x_1, x_2, x_3)$ and may be called the pressure. Show that the equilibrium equations imply that the pressure must satisfy the relation $\nabla p = \mathbf{F}$.

3. A one-dimensional problem of a prismatic bar loaded under its own weight can be modeled by the stress field

$$\sigma_{ij} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

with body forces $F_x = \rho g, F_y = F_z = 0$, where ρ is the mass density and g is the local acceleration of gravity. Using the equilibrium equation, show that the non-zero stress will be given by $\sigma_x = \rho g(l - x)$, where l is the length of the bar.



4. Consider the equilibrium of a two-dimensional differential element in polar coordinates, as shown in the following figure. Explicitly sum the forces and moments and develop the two-dimensional equilibrium equations.