1. For a beam of circular cross-section, analysis from elementary strength of materials theory yields the following stresses:

$$\sigma_x = -\frac{My}{I}, \quad \tau_{xy} = \frac{V\left(R^2 - y^2\right)}{3I}, \quad \sigma_y = \sigma_z = \tau_{xz} = \tau_{yz} = 0,$$

where R is the section radius, I is the moment of inertia, $M(x)$ is the bending moment, and $V(x)$ is the shear force. Assuming zero body forces, investigate that if these stresses satisfy the equilibrium and compatibility equations. The result of this problem is one of many that indicate the approximate nature of strength of materials theory. (*Optional*) *What will happen if the beam is rectangular?*

2. In the absence of body forces, show that the following stresses

$$\sigma_x = xy, \quad \sigma_y = x, \quad \sigma_z = \nu x(1 + y), \quad \tau_{xy} = -\frac{1}{2}y^2, \quad \tau_{xz} = \tau_{yz} = 0$$

satisfy the plane strain formulations, i.e. equilibrium, compatibility, and the relation among the out-of-plane normal stress and two in-plane normal stresses.

3. Derive the polar coordinate strain-displacement relations by using the transformation equations for displacements and strains:

$$u = u_r \cos\theta - u_\theta \sin\theta, \qquad v = u_r \sin\theta + u_\theta \cos\theta;$$
$$\varepsilon_r = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + 2\varepsilon_{xy} \sin\theta\cos\theta,$$
$$\varepsilon_\theta = \varepsilon_x \sin^2\theta + \varepsilon_y \cos^2\theta - 2\varepsilon_{xy} \sin\theta\cos\theta,$$
$$\varepsilon_{r\theta} = -\varepsilon_x \sin\theta\cos\theta + \varepsilon_y \sin\theta\cos\theta + \varepsilon_{xy}\left(\cos^2\theta - \sin^2\theta\right).$$

4. For rigid-body motion, the strains will vanish. Under these conditions, integrate the strain-displacement relations you just derived from the previous problem to show that the most general form of a rigid-body motion displacement field in polar coordinates is given by

$$u_r{*} = a\sin\theta + b\cos\theta,$$
$$u_\theta{*} = a\cos\theta - b\sin\theta + cr,$$

where a, b, c are constants. Also show that this result is consistent with the Cartesian form of rigid-body motion.