Cyclic Loading and Fatigue

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Cyclic Stress and Fatigue

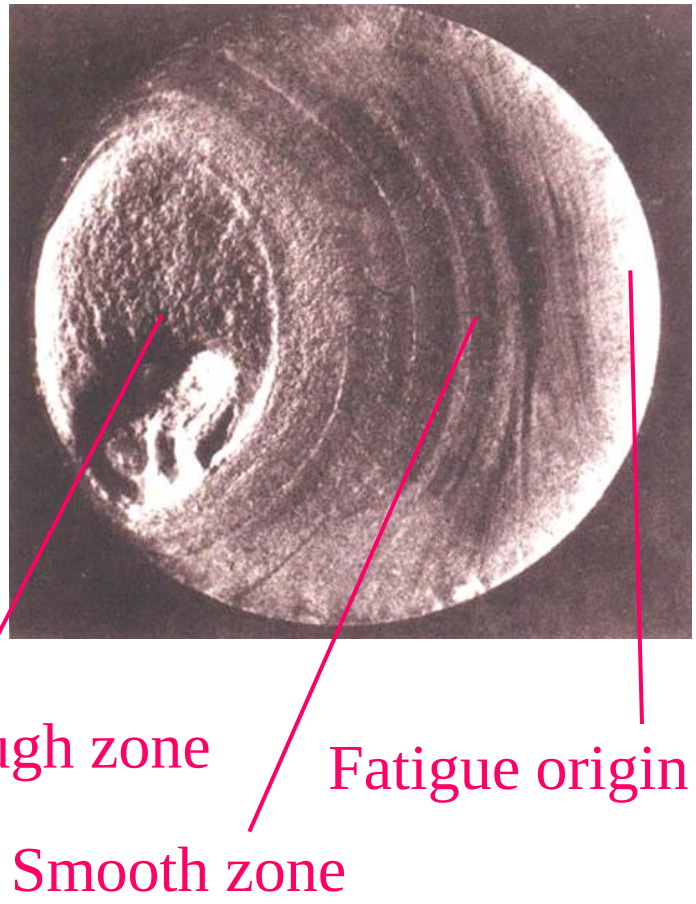
- **Cyclic Stress:** stresses varying periodically with time.

- **Fatigue Failure:** failure of structural members under the application of cyclic stresses typically with magnitude far less than yield stress.

- **Examples:** (a) coupling gears; (b) eccentric motors; (c) train wheel axes.

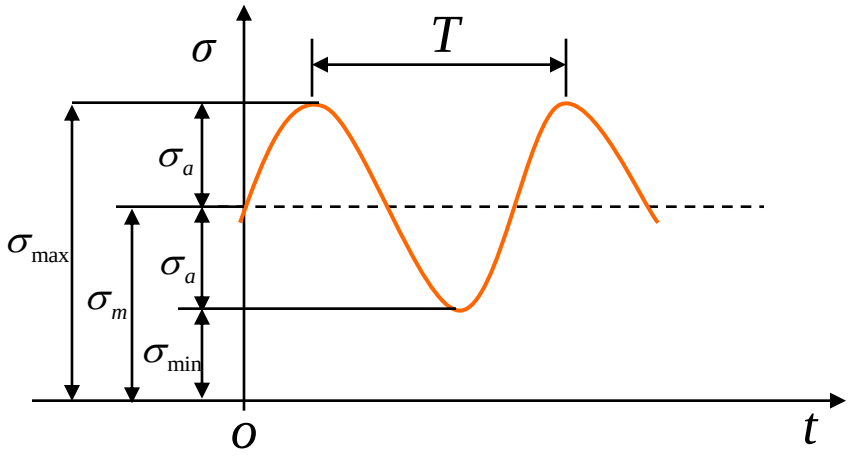
Characteristics of Fatigue Failure

- Fatigue failure stress << ultimate strength / yield stress
- Fatigue failure typically occurs after many cycles of stress application
- Fatigue failure experiences three stages: crack initiation & propagation, and eventual fracture
- Fatigue failure happens in the form of brittle fracture
- Fracture surface is composed of a rough zone and a smooth zone



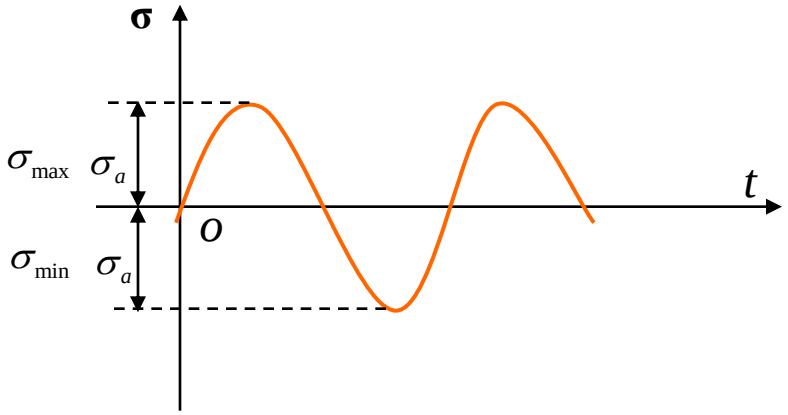
Technique Terms Involved in Cyclic Loading

- Maximum Stress: $\sigma_{\max}$
- Minimum Stress: $\sigma_{\min}$
- Average Stress: $\sigma_m = \dfrac{\sigma_{\min} + \sigma_{\max}}{2}$
- Stress Amplitude: $\sigma_a = \dfrac{\sigma_{\max} - \sigma_{\min}}{2}$
- Stress Scope: $\Delta\sigma = \sigma_{\max} - \sigma_{\min}$
- Cycle Characteristics: $r = \dfrac{\sigma_{\min}}{\sigma_{\max}}$



Classification of Cyclic Loading

- Symmetric Cycling: equal but opposite maximum and minimum stress



$$\sigma_a = \sigma_{max} = |\sigma_{min}|$$

$$\sigma_m = 0$$

$$r = \frac{\sigma_{min}}{\sigma_{max}} = -1$$

- Unsymmetric Cycling

$$\sigma_{max} = \sigma_m + \sigma_a$$

$$\sigma_{min} = \sigma_m - \sigma_a$$

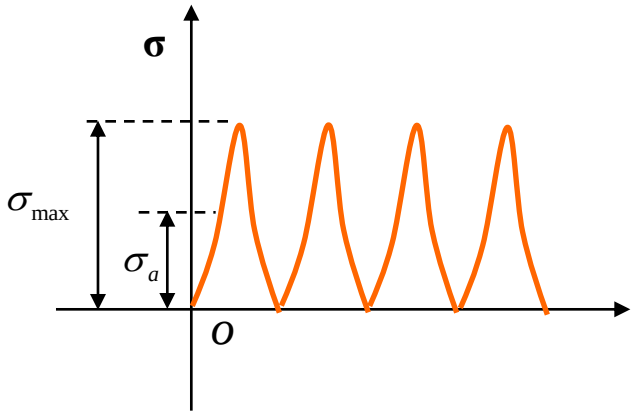
$$r = \frac{\sigma_{min}}{\sigma_{max}}$$

Classification of Cyclic Loading

- Pulse Cycling

$$\sigma_{\max} = \sigma_{\max} \qquad \sigma_{\min} = 0$$

$$\sigma_a = \frac{\sigma_{\max}}{2} = \sigma_m \qquad r = 0$$



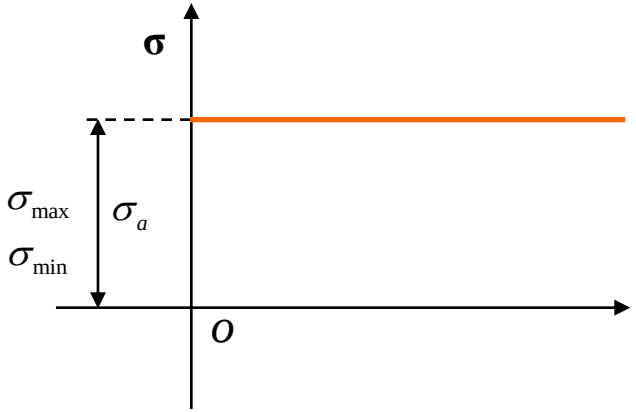
- Or:

$$\sigma_{\max} = 0 \qquad \sigma_{\min} = \sigma_{\max} \qquad \sigma_a = \frac{\sigma_{\min}}{2} = \sigma_m \qquad r = -\infty$$

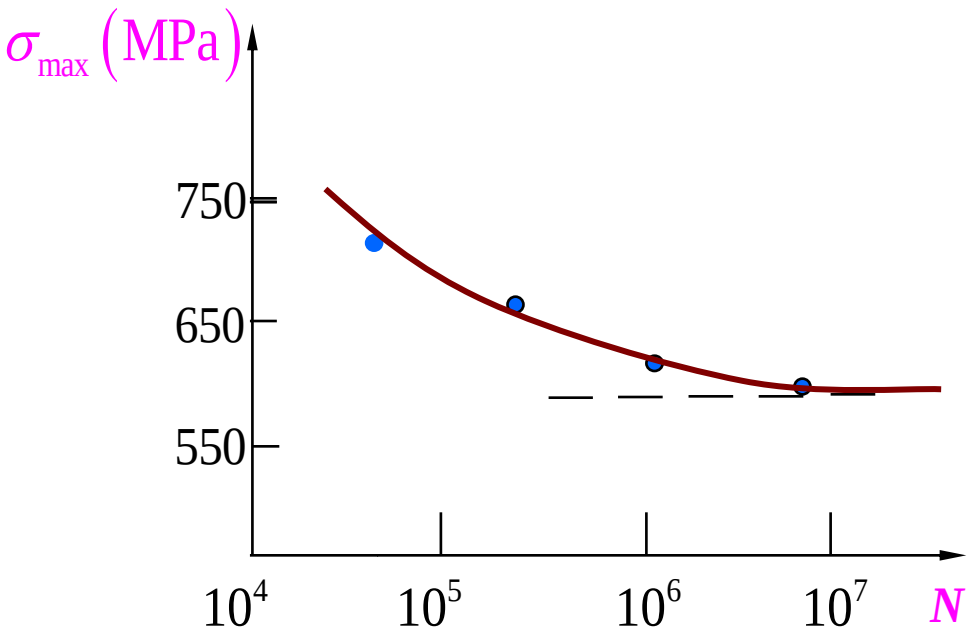
- Static (Constant) Stress

$$\sigma_{\max} = \sigma_{\min} = \sigma_m$$

$$\sigma_a = 0 \qquad r = 1$$



Stress-life (S-N) Diagram



- Fatigue properties are shown on σ-N diagrams.
- A member may fail due to *fatigue* at stress levels significantly below the ultimate strength if subjected to many loading cycles.
- When the stress is reduced below the ***endurance limit*** **(σ_r)**, fatigue failures do not occur for any number of cycles.

Factors Affect Fatigue Strength

- Stress Concentration
- Surface Roughness
- Surface Strength

Allowable Stress Scope for Equal-amplitude Fatigue

- Equal-amplitude:

$$\Delta\sigma = 2\sigma_a = \sigma_{\max} - \sigma_{\min} = \text{const.}$$

- Experimental data shows:

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} = \left(a/N\right)^{1/\beta}$$
$$\Rightarrow \left[\Delta\sigma\right] = \left(C/N\right)^{1/\beta}$$

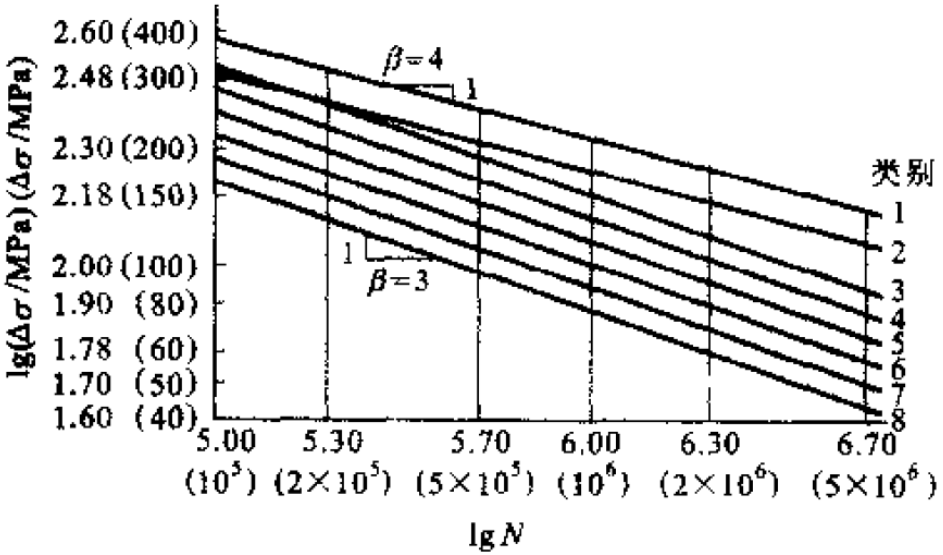


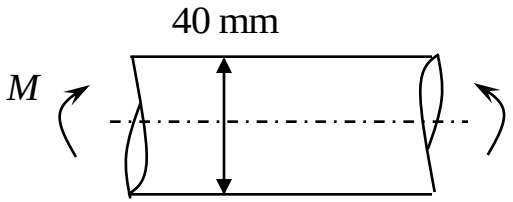
表 6－1　参数 C、β 值

构件和连接类别	1	2	3	4
C	1940×10^{12}	861×10^{12}	3.26×10^{12}	2.18×10^{12}
β	4	4	3	3
构件和连接类别	5	6	7	8
C	1.47×10^{12}	0.96×10^{12}	0.65×10^{12}	0.41×10^{12}
β	3	3	3	3

- Fatigue strength condition: $\left[\Delta\sigma\right] \le n_f \Delta\sigma.$

Sample Problem

- Given: $M_{\max} = 5\,M_{\min} = 512$ N·m, $n_s = 2$, $\sigma_Y = 540$ Mpa, $n_f = 2$, $C = 2.18\times10^{12}$, $\beta = 3$. Examine both the static and fatigue strength condition for $N = 2\times10^6$.



- Solution:

1. Stress scope for equal-amplitude fatigue:

$$\sigma_{\max} = \frac{M_{\max}}{W_z} = \frac{512}{\pi D^3/32} = 81.5\ \text{MPa}; \qquad \sigma_{\min} = \frac{1}{5}\sigma_{\max} = 16.3\ \text{MPa}$$

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} = 65.2\ \text{MPa}$$

2. Fatigue strength check: $[\Delta\sigma] = \left(\frac{C}{N}\right)^{1/\beta} = \left(\frac{2.18\times10^{12}}{2\times10^6}\right)^{1/3}\ \text{MPa} = 102.9\text{MPa}$

$$\frac{[\Delta\sigma]}{\Delta\sigma} \approx 1.58 < n_f = 2 \qquad (\text{NG.})$$

3. Static strength check: $\frac{\sigma_Y}{\sigma_{\max}} = 6.62 > n_s = 2 \quad (\text{OK.})$

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