Bending Internal Forces

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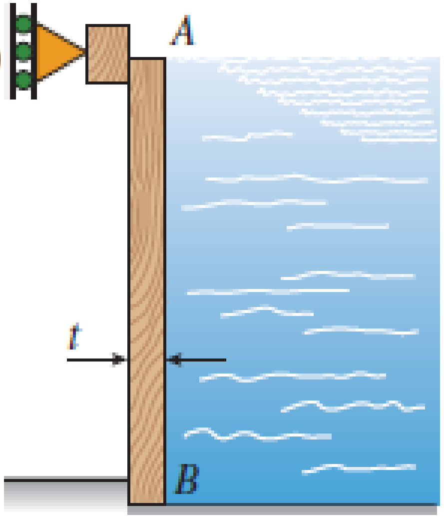
- Diagram of Internal Forces for Plane Frames（平面刚架的内力图）
- Diagram of Internal Forces for Curved Beams（曲梁的内力图）

Introduction





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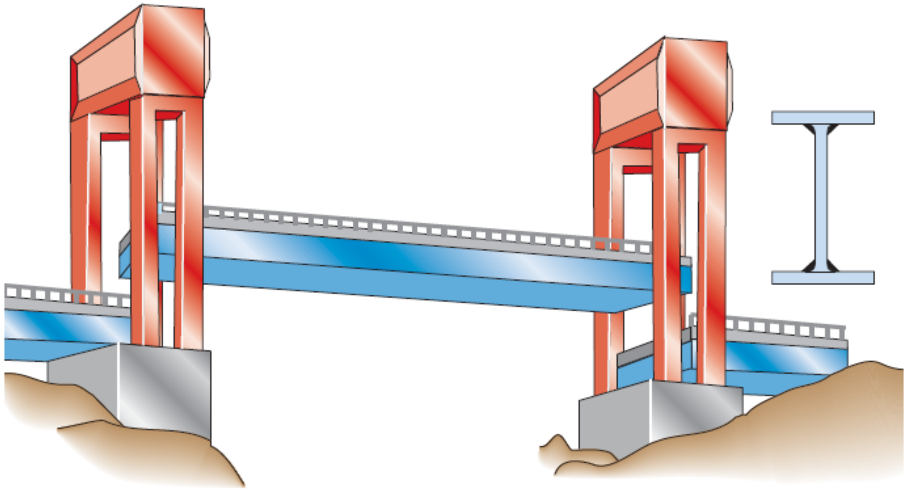

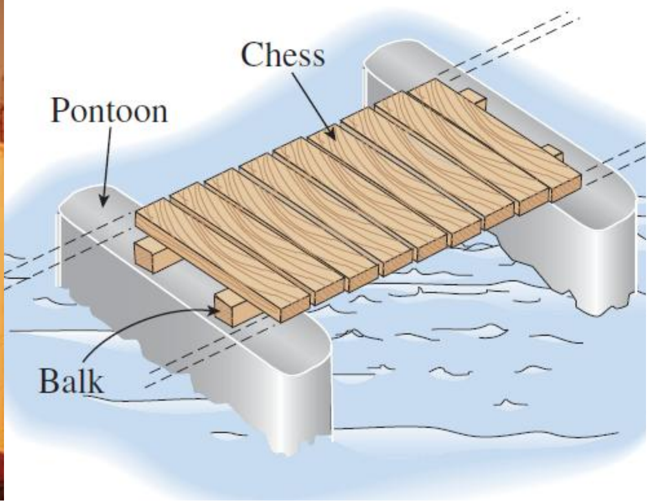

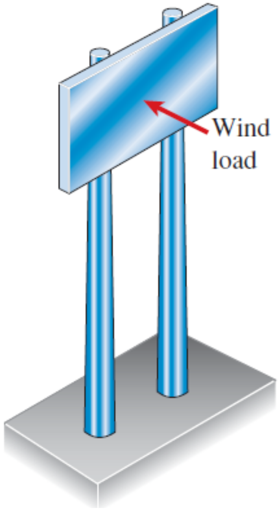

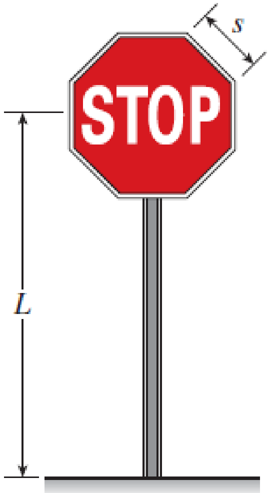

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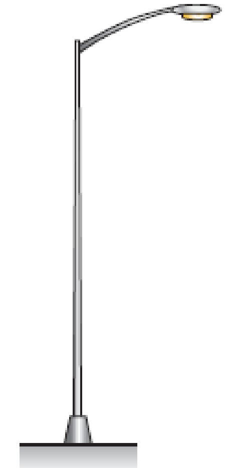

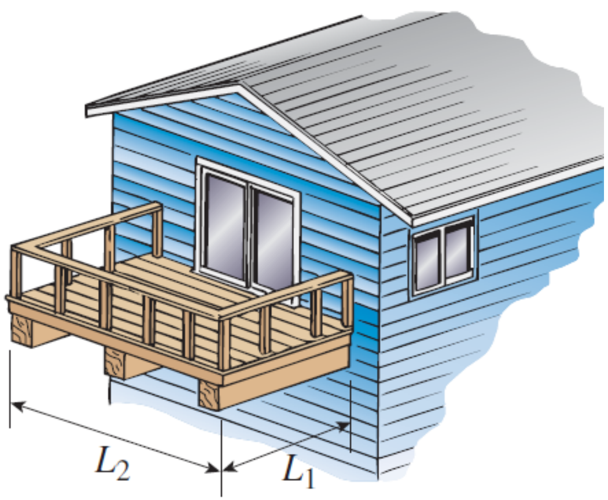

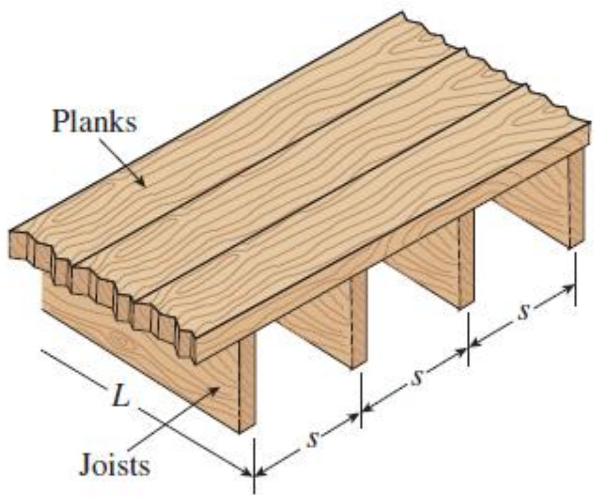

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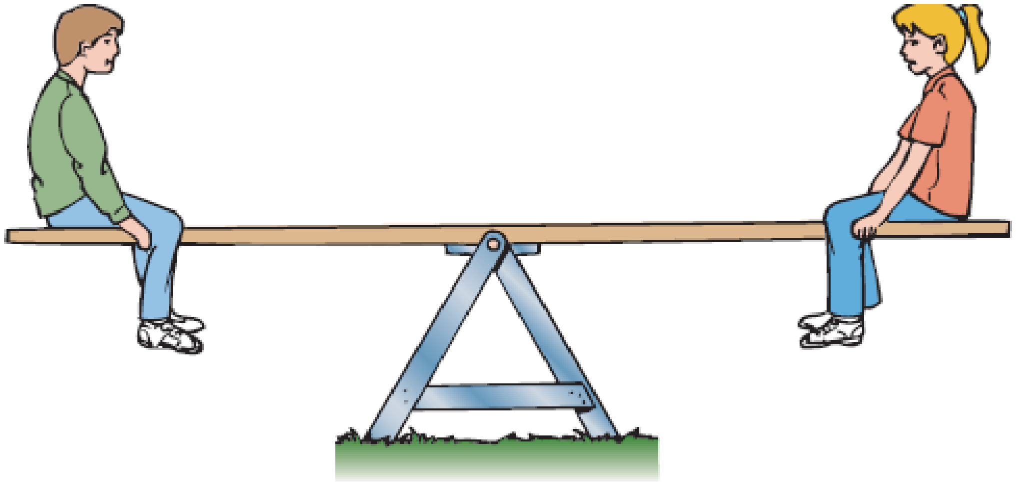

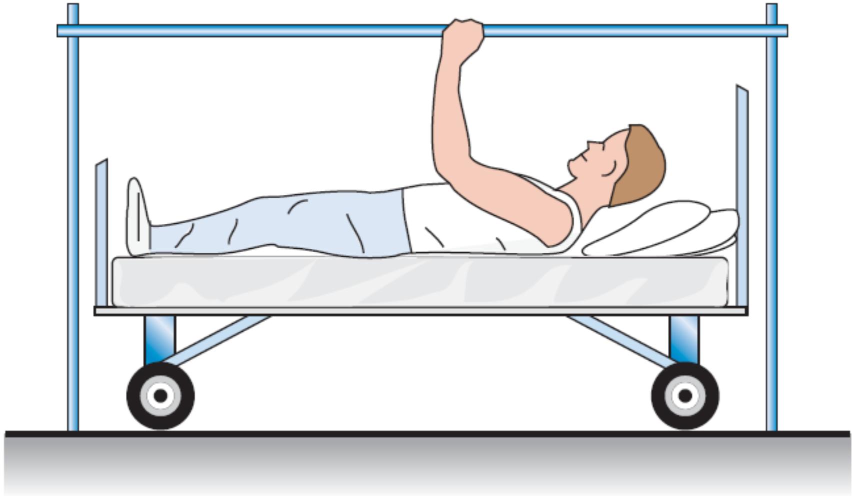

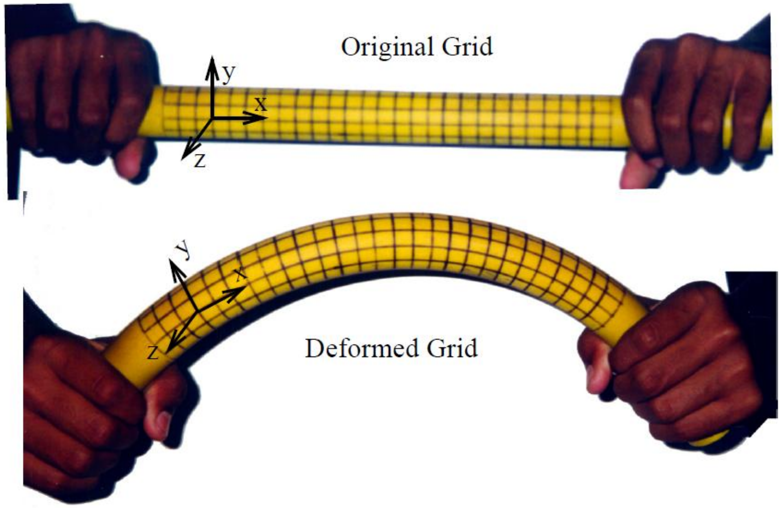

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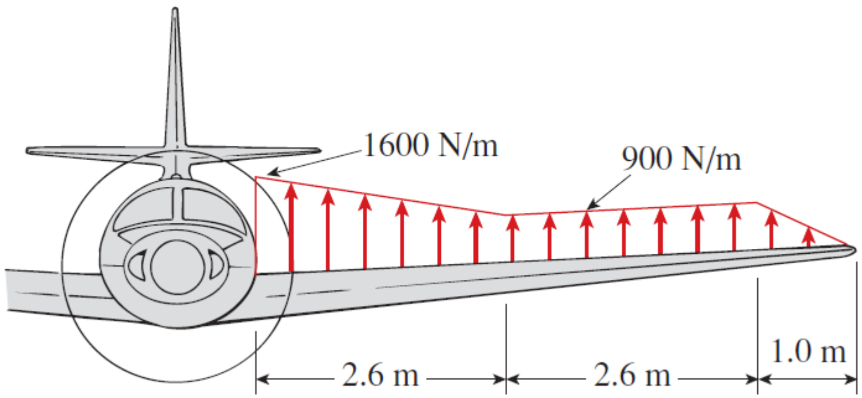

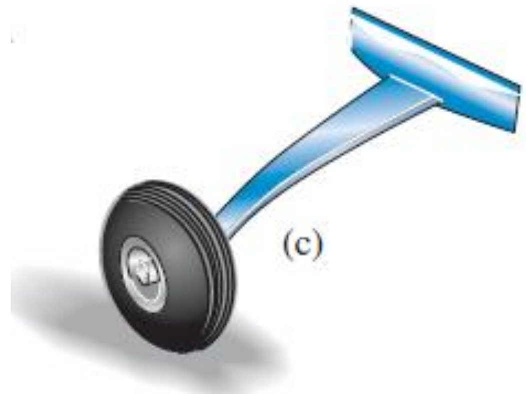

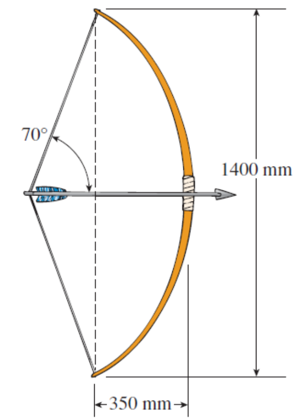

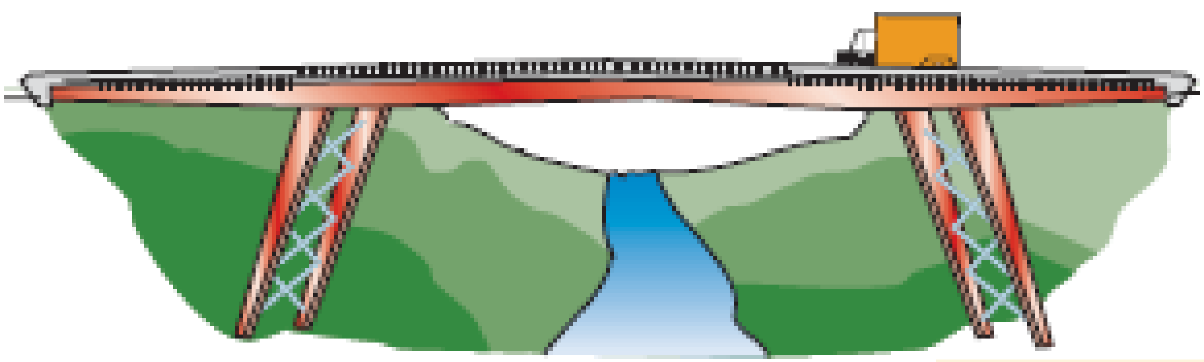

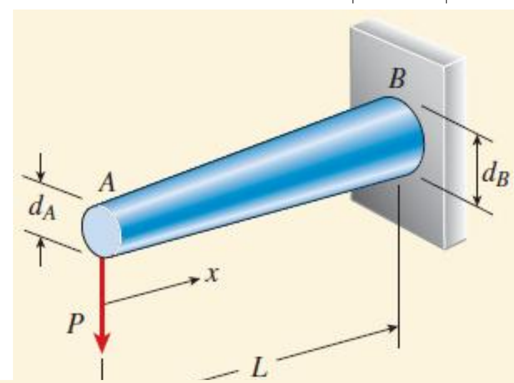

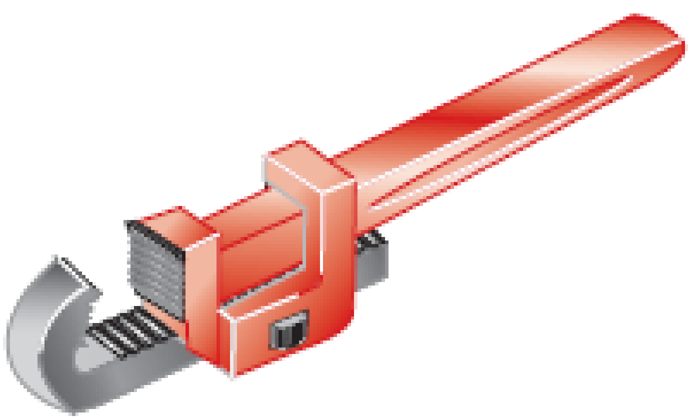

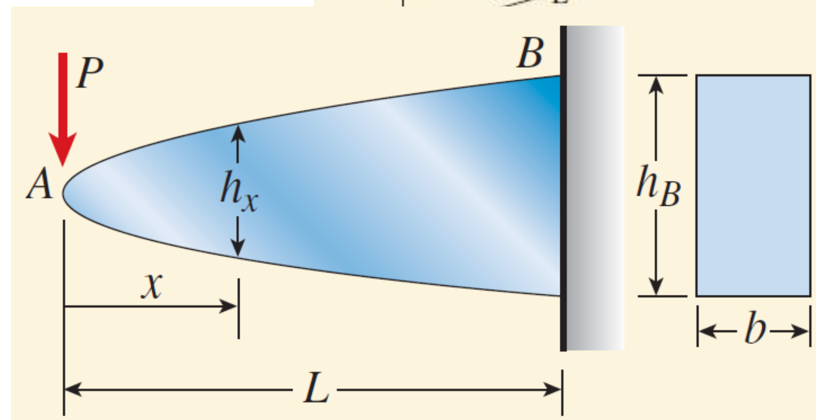

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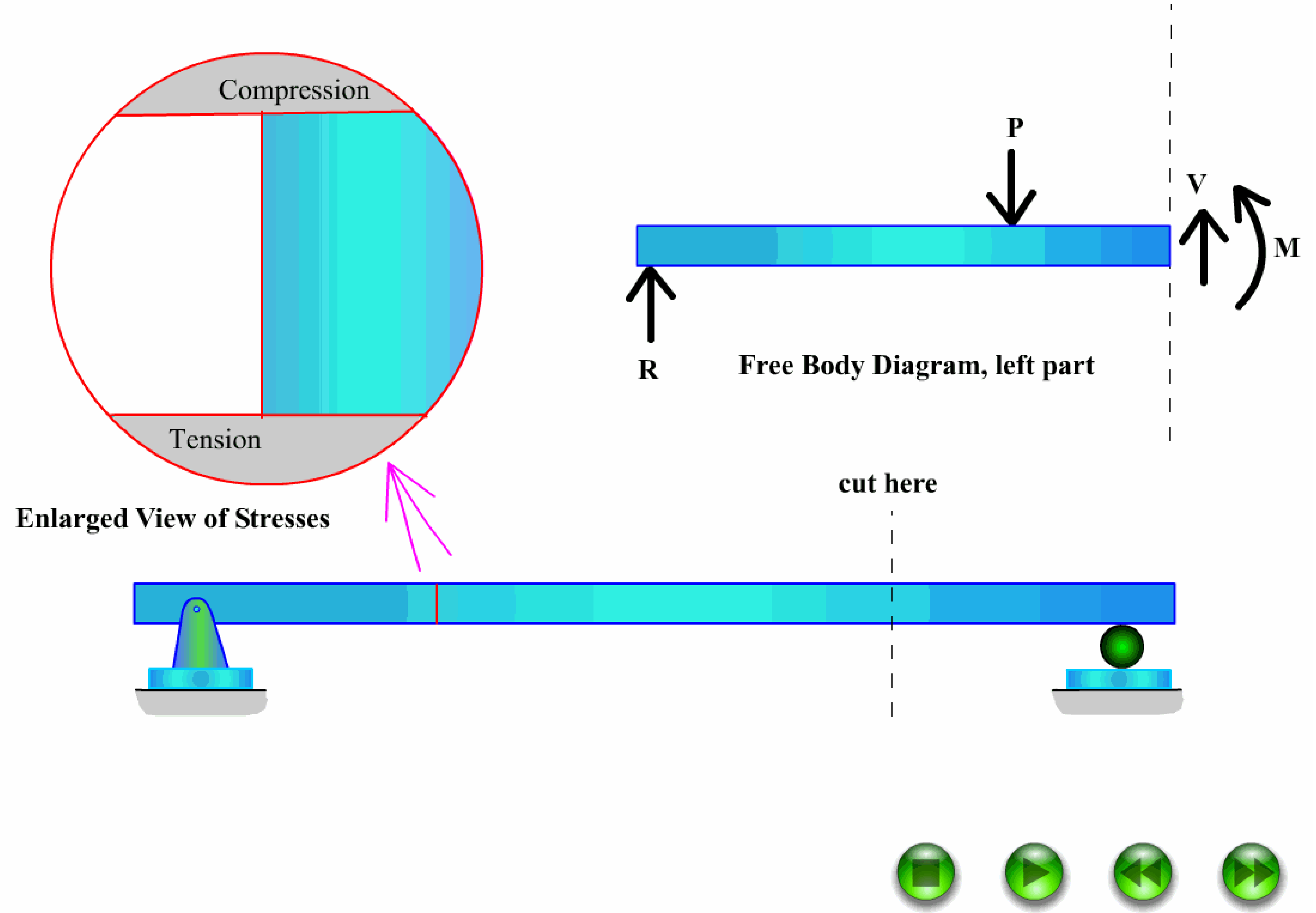
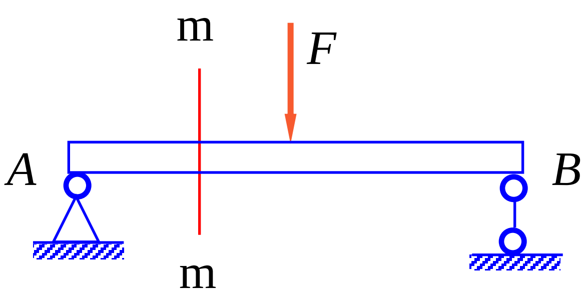
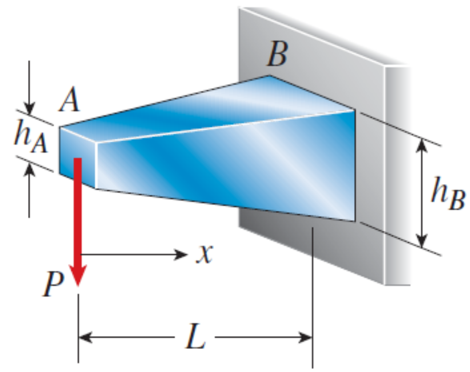
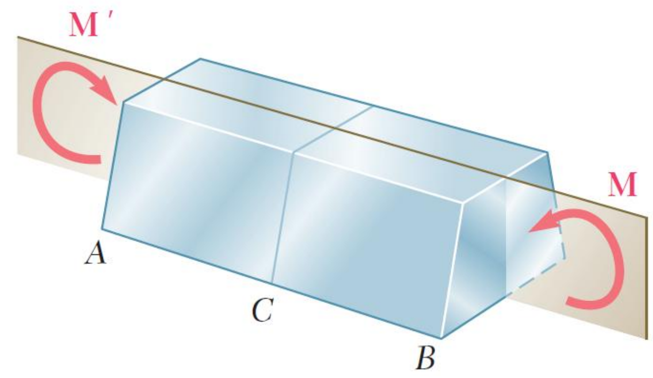


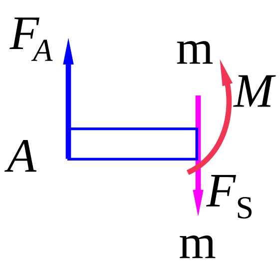
The Form of Internal Forces in Beams

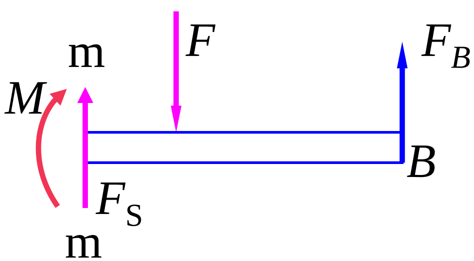
- Internal forces on cross-sections of bending beams





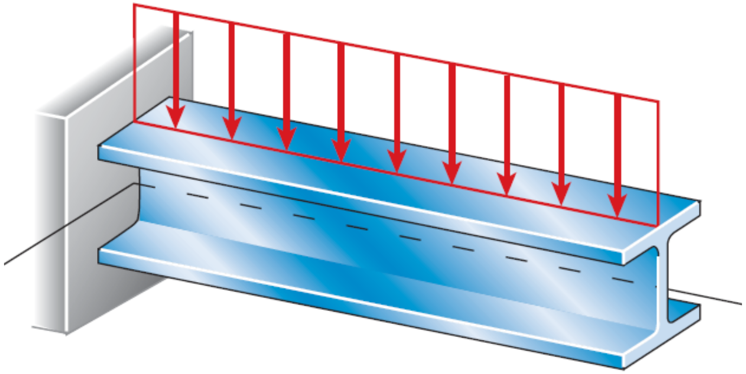


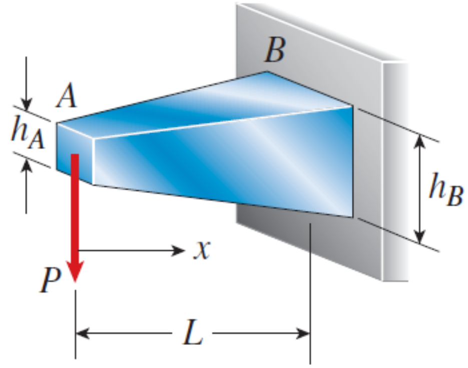


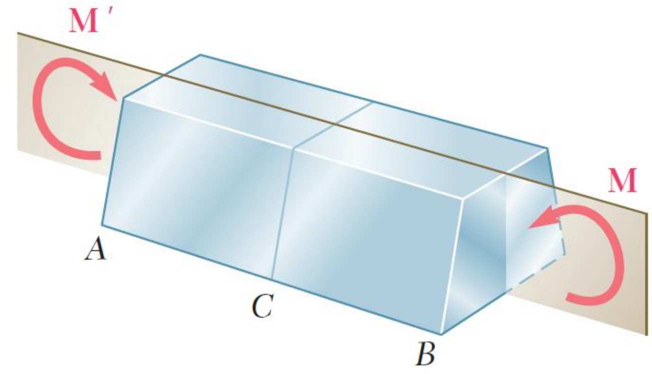


The internal forces on cross-section m-m can be simplified as *shearing forces* and *bending moments* at cross-sectional centroid.

Symmetric Bending





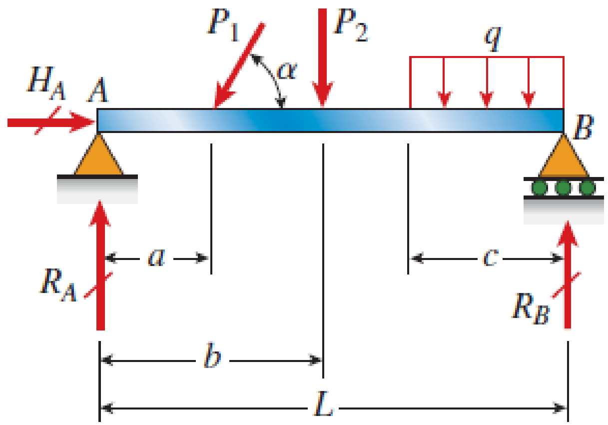


- **Beam:** one type of prismatic bars which takes bending as the main form of deformation.

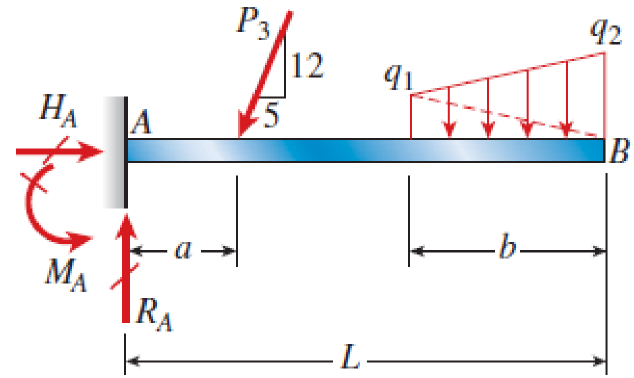
- **Symmetric longitudinal plane:** formed by the centroidal axes of cross sections.

- **Symmetric bending:** all external forces & moments acting in the symmetric longitudinal plane (beam axis being bent from a straight line to a curve within the symmetric longitudinal plane).

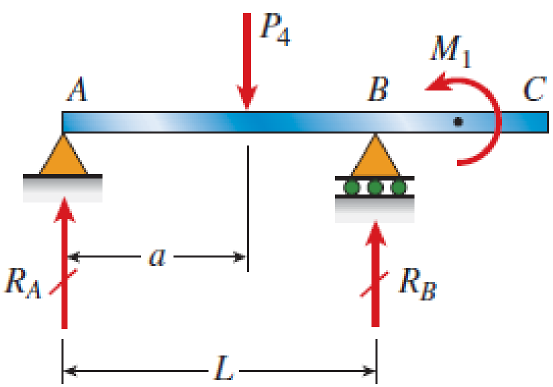
Types of Beams



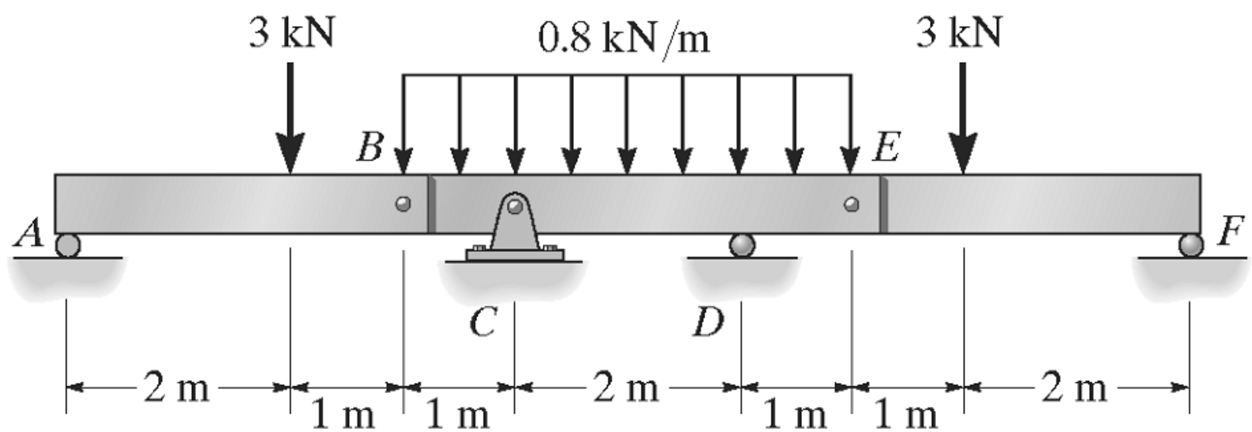
Simply supported beam



Cantilever beam

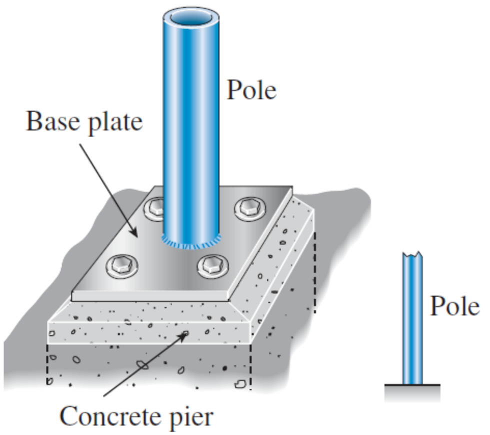


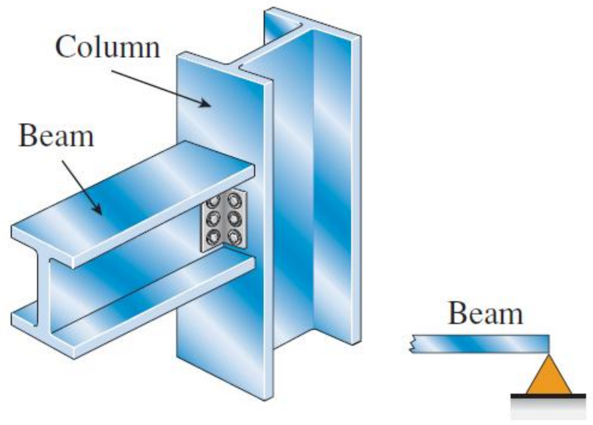
Beam with an overhang

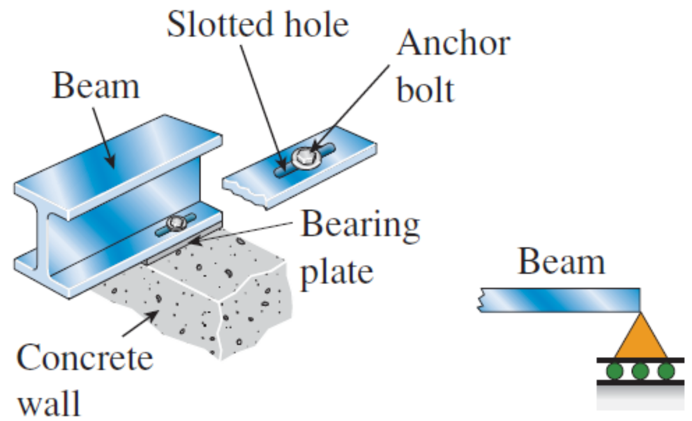


Compound beam

Types of Supports and Reactions



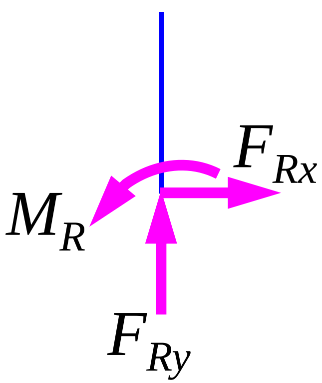


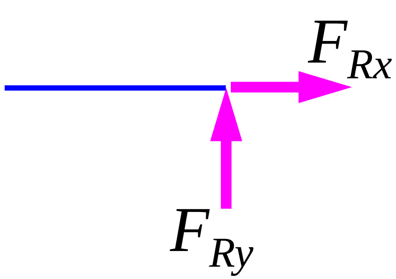


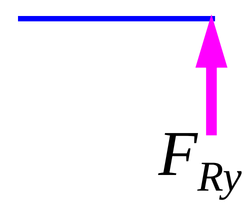
Fixed support

Hinged or pinned support

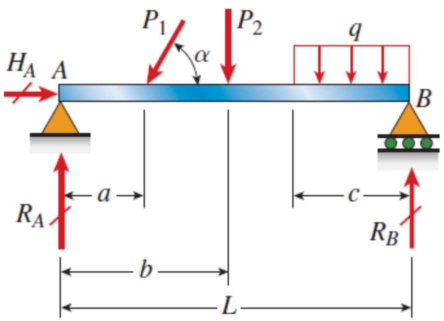
Roller support

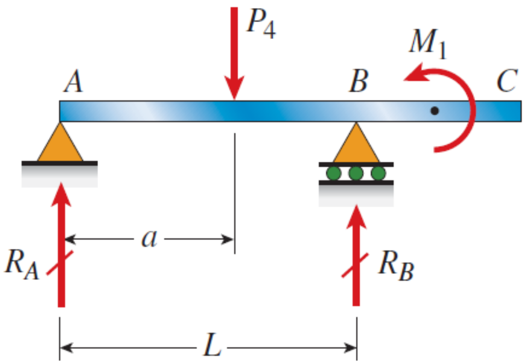


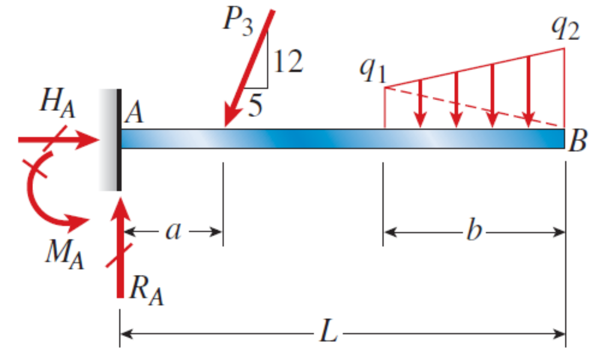




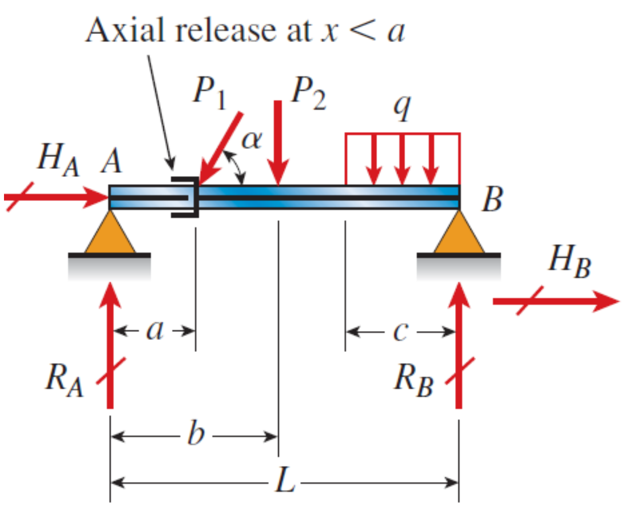
Internal Releases

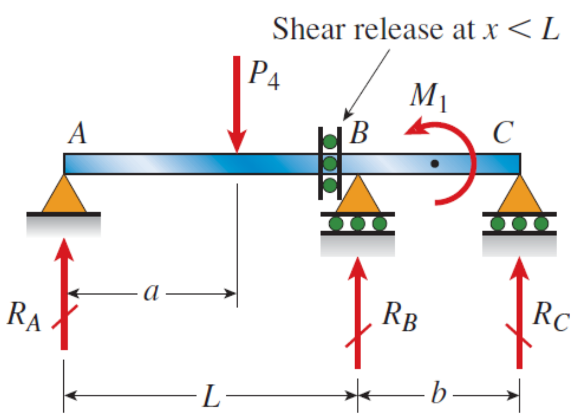


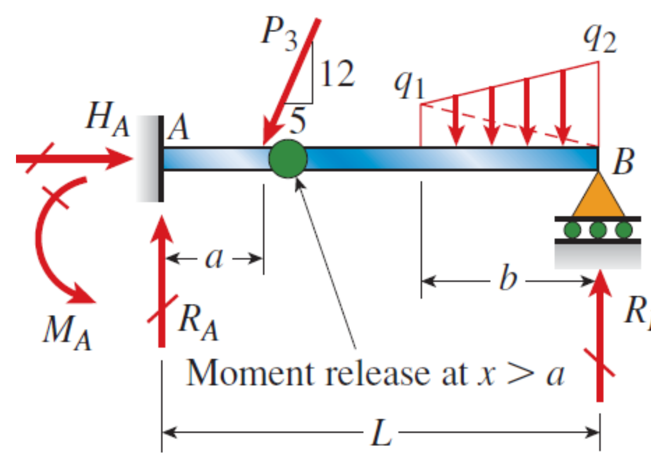




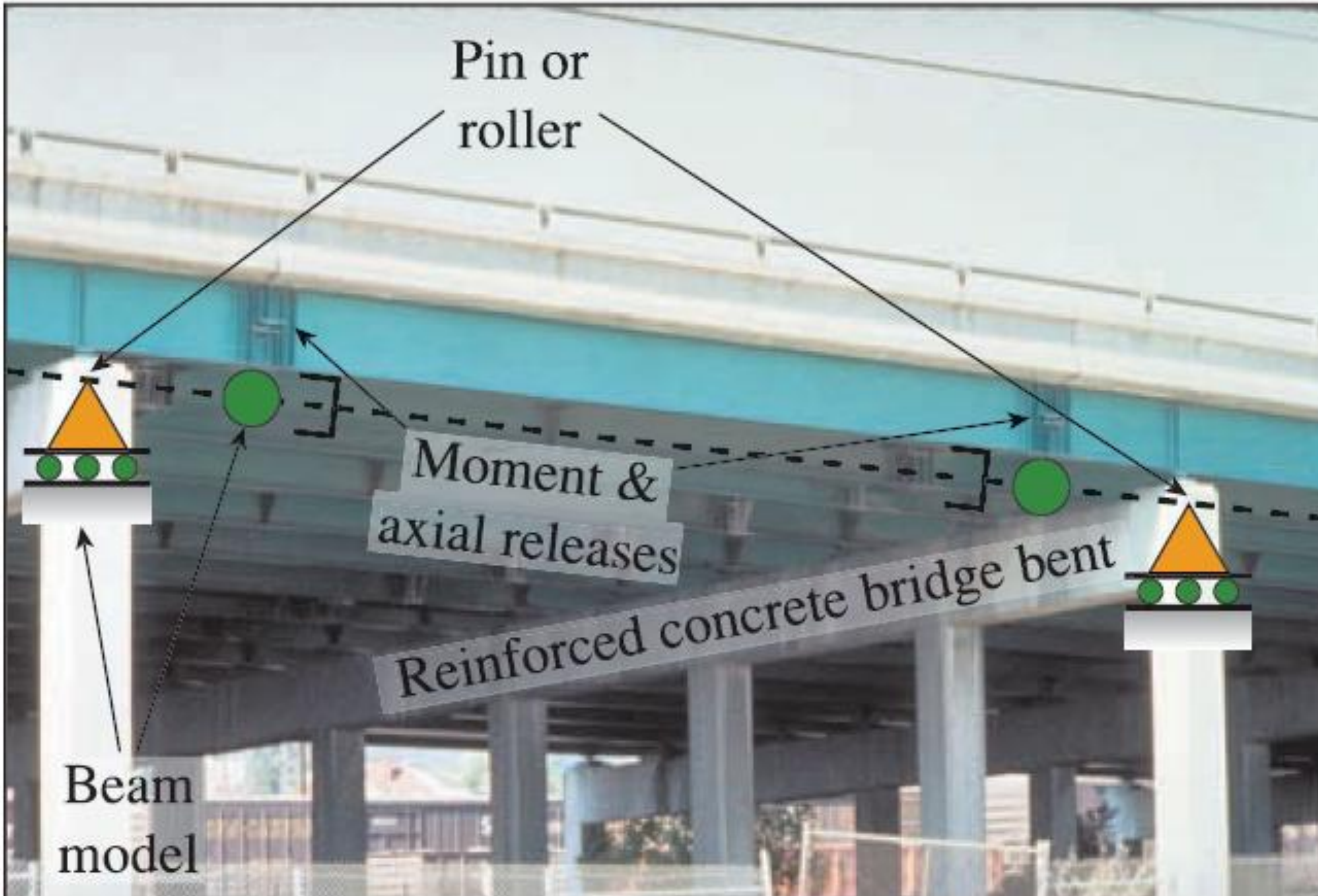
- Axial release
- Shear release
- Moment release





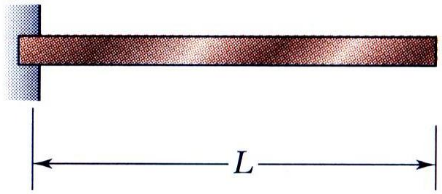


Internal Releases

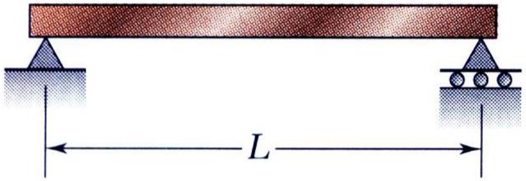


Determinacy of Beams

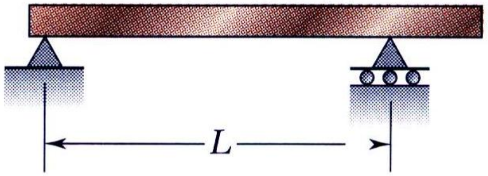
- Statically determinate beams



- Cantilever beam

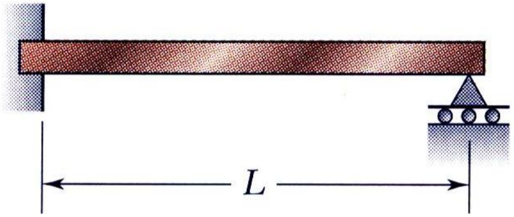


- Simply supported beam

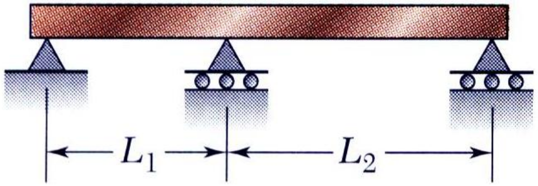


- Overhanging beam

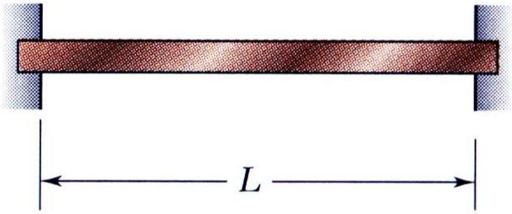
- Statically indeterminate beams



- Simply supported cantilever beam



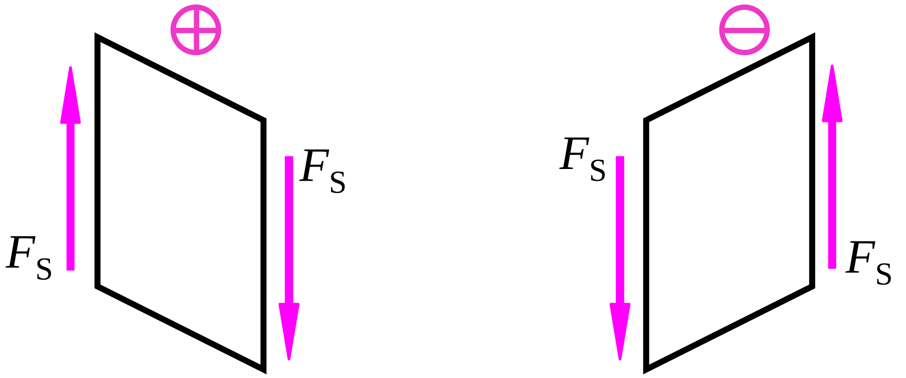
- Continuous beam



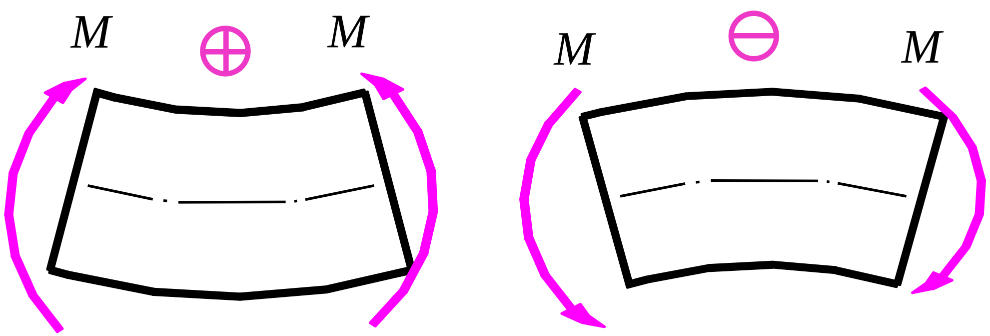
- Fixed beam

Sign Convention of Shearing Forces and Bending Moments

- Shearing forces
  - Positive: left up / right down (clockwise loop)

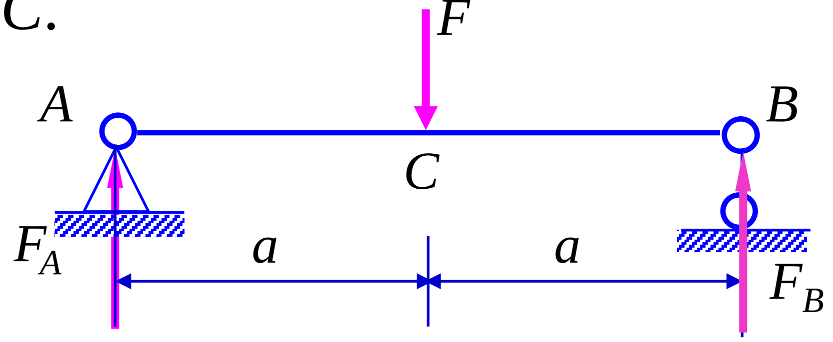


- Bending moment
  - Positive: upper half under compression / lower half under tension (top concave / bottom convex)



Sample Problem

- Find the shearing force and bending moment at the left and right cross-section at point *C*.

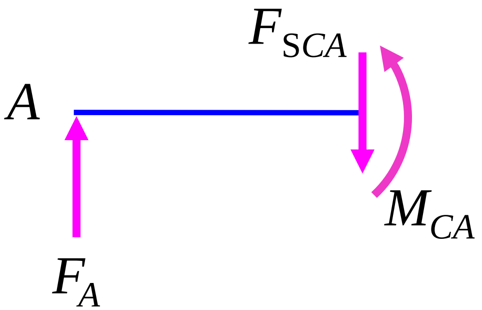


- Solution:

1. Reaction forces at the supports

$$\boldsymbol{F_A = F_B = \frac{F}{2}}$$

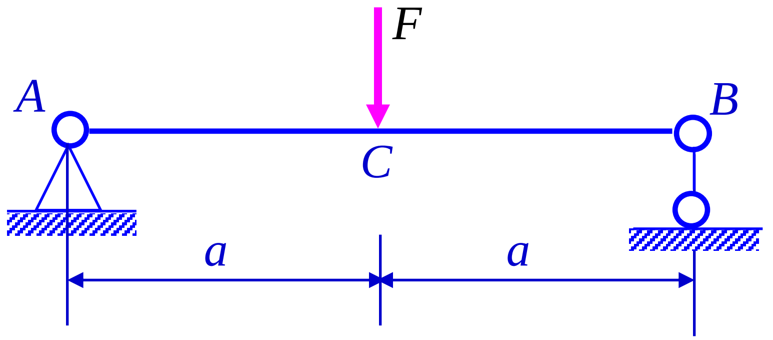
2. Internal forces (assuming positive) at the right cross-section of *C*.



$$F_{SCA} = F_A = \frac{F}{2}$$

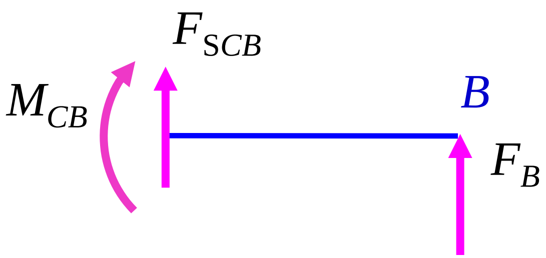
$$M_{CA} = F_A \cdot a = \frac{Fa}{2}$$

3. Internal forces (assuming positive) at the left cross-section of *C*.



$$F_{SCB} = -F_B = -\frac{F}{2}$$

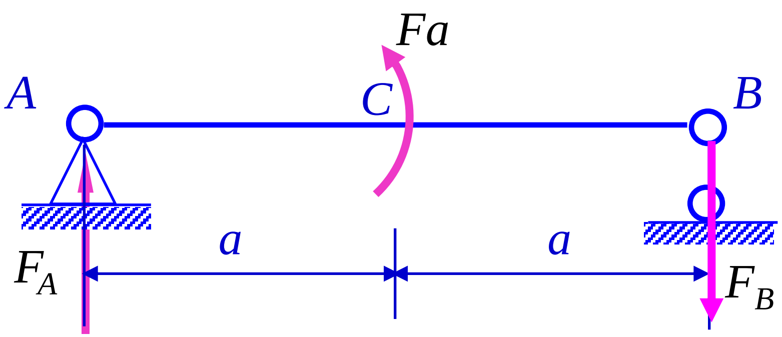
$$M_{CB} = F_B \cdot a = \frac{Fa}{2}$$



Note: for a concentrated force acting at *C*, the change of bending moment is zero while that of shearing force is equal to the magnitude of the concentrated force.

Sample Problem

- Find the shearing force and bending moment at the left and right cross-section at point *C*.

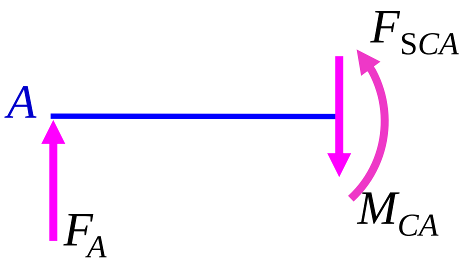


- Solution:

1. Reaction forces at the supports

$$\boldsymbol{F_A = F_B = \frac{F}{2}}$$

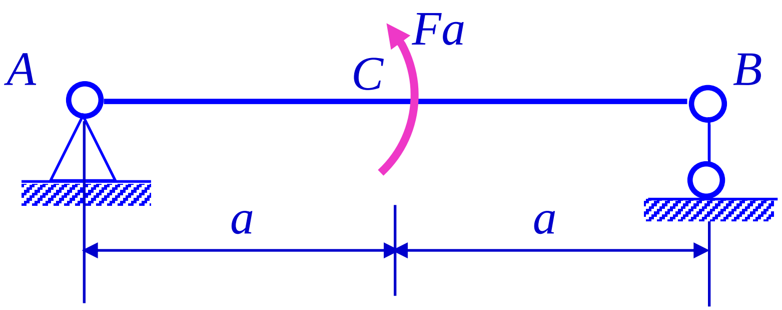
2. Internal force at the right cross-section of *C*.



$$F_{SCA} = F_A = \frac{F}{2}$$

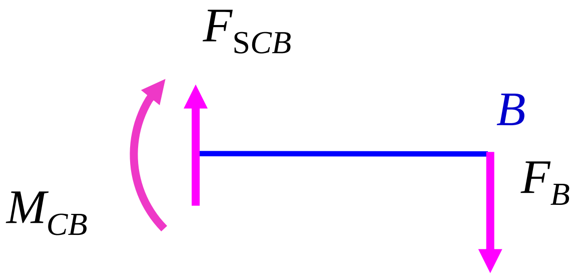
$$M_{CA} = F_A \cdot a = \frac{Fa}{2}$$

3. Internal force at the left cross-section of C.



$$F_{SCB} = F_B = \frac{F}{2}$$

$$M_{CB} = -F_B \cdot a = -\frac{Fa}{2}$$



Note: For a concentrated moment acting at *C*, the change of shearing force is zero while that of bending moment is equal to the magnitude of the concentrated moment.

Direct Calculation of Shearing Forces

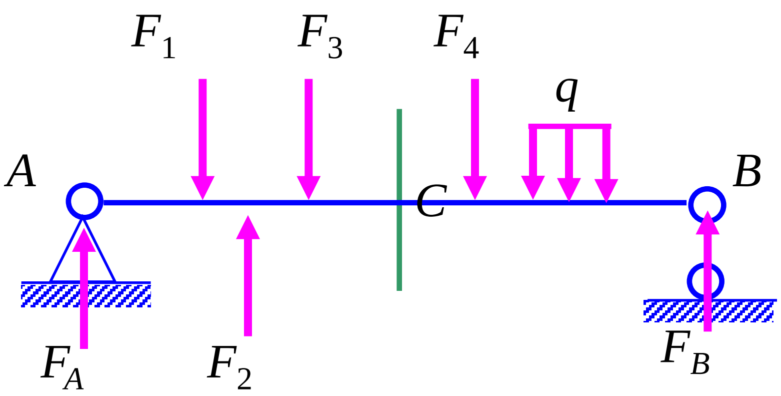
- The shearing forces at a beam cross-section is equal to the net of the external forces collected from either side of the cross-section.

- Upward net from left portion or downward net from right portion results in positive shearing force at the cross-section.

- All forces acting on a beam, including the reaction forces at the supports, must be taken into consideration.

Sample Problem

- Directly calculate the shearing force at cross-section *C* without the explicit use of method of section.



- Solution:
  - Based on the left portion from cross-section *C*:

$$F_{SC} = F_A - F_1 + F_2 - F_3$$

  - Based on the right portion from cross-section *C*:

$$F_{SC} = -F_B + ql + F_4$$

Note: in practice, we pick the portion with simpler loadings.

Direct Calculation of Bending Moments

- The bending moment at a beam cross-section is equal to the net of the external moments with respect to the cross-section centroid, collected from either side of the cross-section

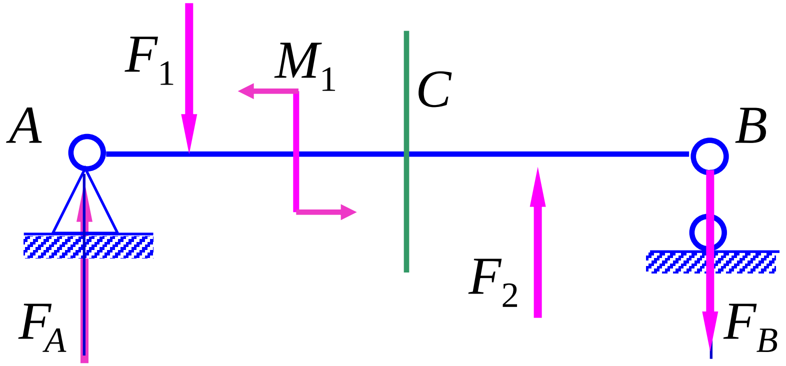
- Clockwise net from left portion or counter clockwise net from right portion results in positive bending moment.

- All forces acting on a beam, including the reaction forces at the supports, must be taken into consideration.

Sample Problem

- Directly calculate the bending moment at cross-section *C* without the explicit use of method of section.

- Solution:



  - Based on the left portion from cross-section *C*

$$M_C = F_A d_A - F_1 d_1 - M_1$$

  - Based on the right portion from cross-section C

$$M_C = -F_B d_B + F_2 d_2$$

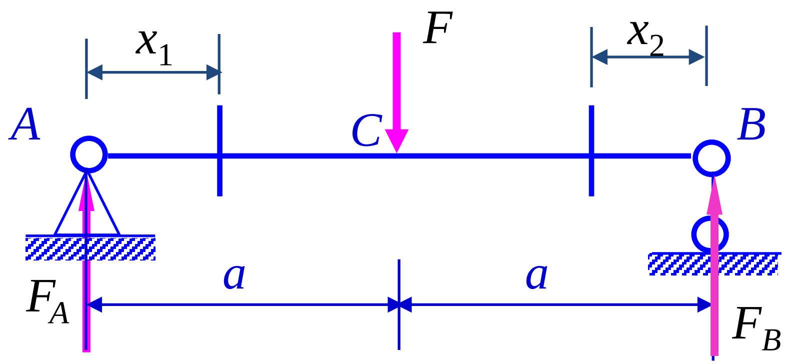
Diagram of Shearing Forces & Bending Moments

- Equation of shearing forces and bending moments
  - Equation of shearing forces: $F_S = F_S(x)$
  - Equation of bending moments: $M = M(x)$
  - x: denotes the position of cross-section.

- Diagram of shearing forces and bending moments
  - Abscissa: cross-section position (x)
  - Ordinate: shearing forces (F_S) / bending moments (M)

Sample Problem

- Draw the diagram of shearing forces and bending moments



- Solution:

1. Reaction force at the supports: $\boldsymbol{F_A = F_B = \frac{F}{2}}$

2. Equation of internal forces

$$F_S(x_1) = F_A = \frac{F}{2} \qquad \boldsymbol{x_1 \in [0, a)}$$

Positive: left & upward

$$F_S(x_2) = -F_B = -\frac{F}{2} \qquad \boldsymbol{x_2 \in [0, a)}$$

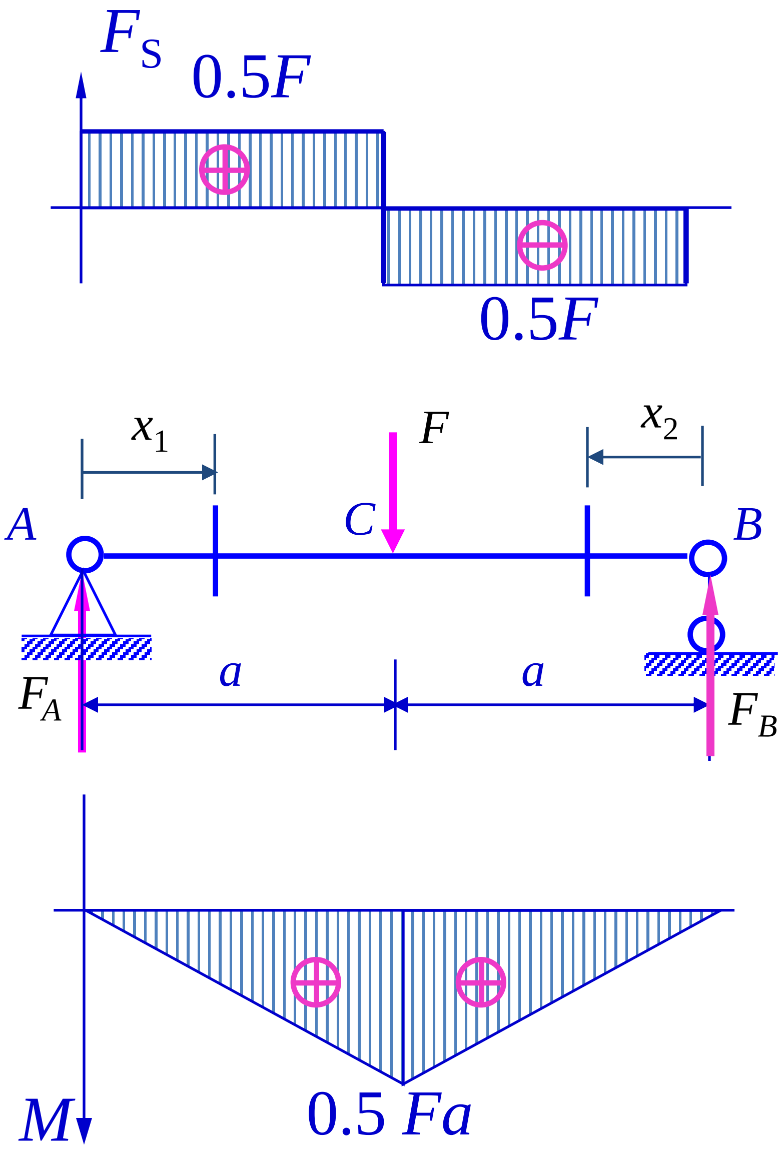
Negative: right & upward

$$M(x_1) = F_A x_1 = \frac{Fx_1}{2} \quad x_1 \in [0,a)$$

Positive: left & clockwise

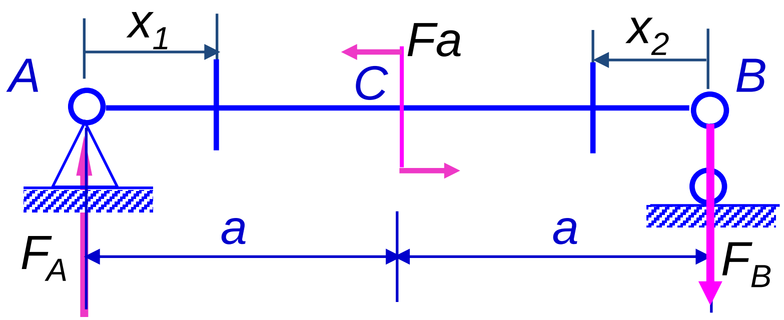
$$M(x_2) = F_B x_2 = \frac{Fx_2}{2} \quad x_2 \in [0,a)$$

Positive: right & counter clockwise



Sample Problem

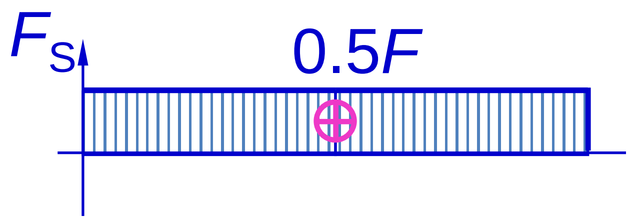
- Draw the diagram of shearing forces and bending moments



- Solution:

1. Reaction forces at the supports

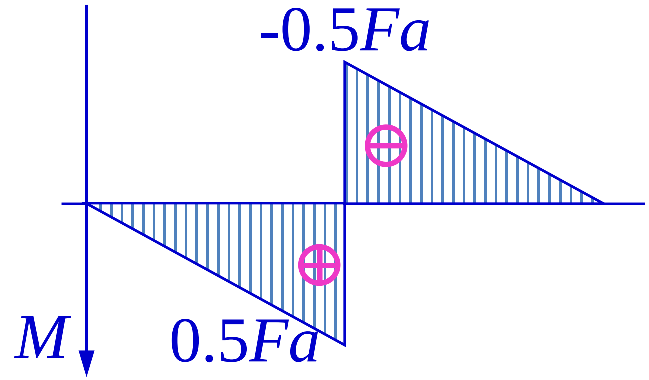
$$F_A = F_B = \frac{F}{2}$$



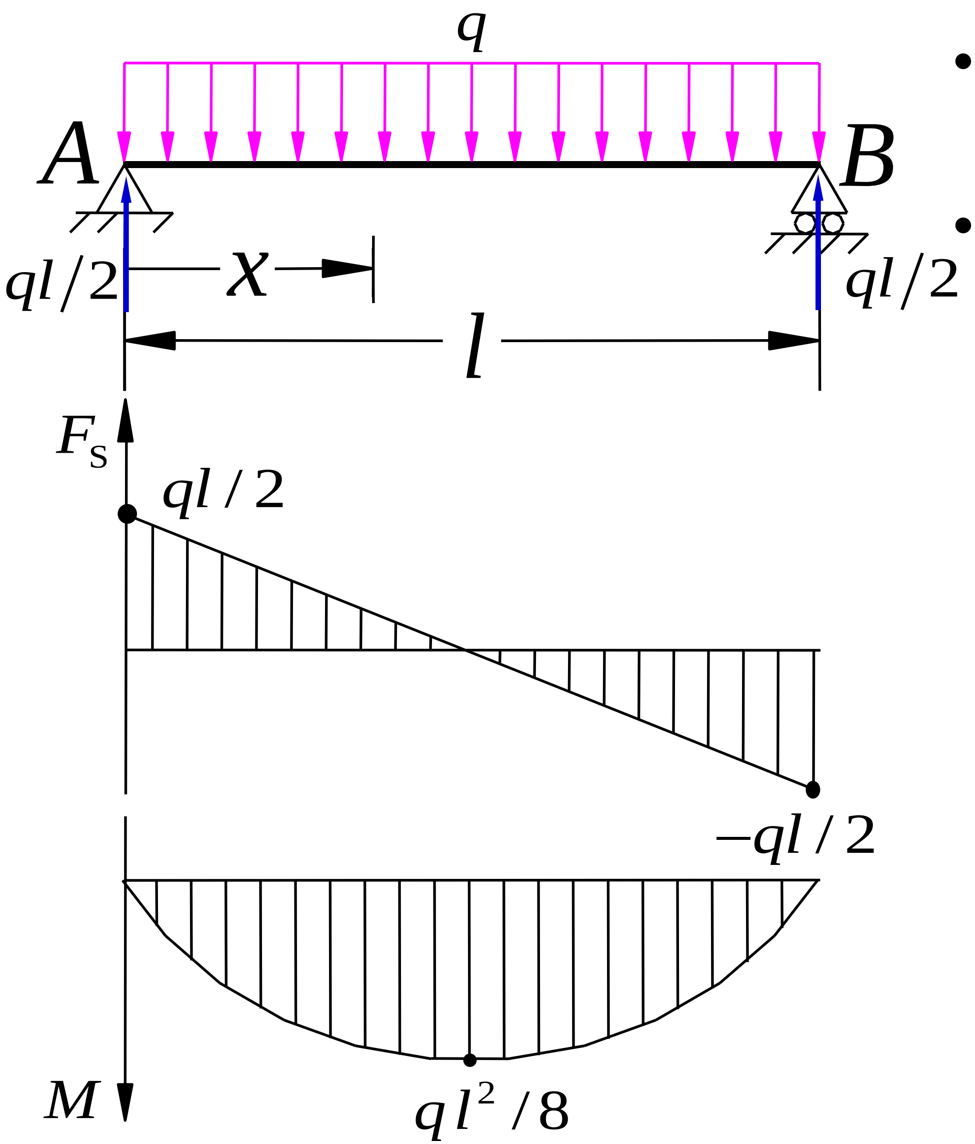
2. Equations of internal forces

$$F_S(x_1) = \frac{F}{2}; \quad F_S(x_2) = \frac{F}{2}$$

$$M(x_1) = \frac{Fx_1}{2}; \; M(x_2) = -\frac{Fx_2}{2}$$



Sample Problem



- Plot the diagram of shearing forces and bending moments
- Solution:

1. Reaction forces at the supports

2. Equation of internal forces

$$F_S(x) = \frac{ql}{2} - qx$$

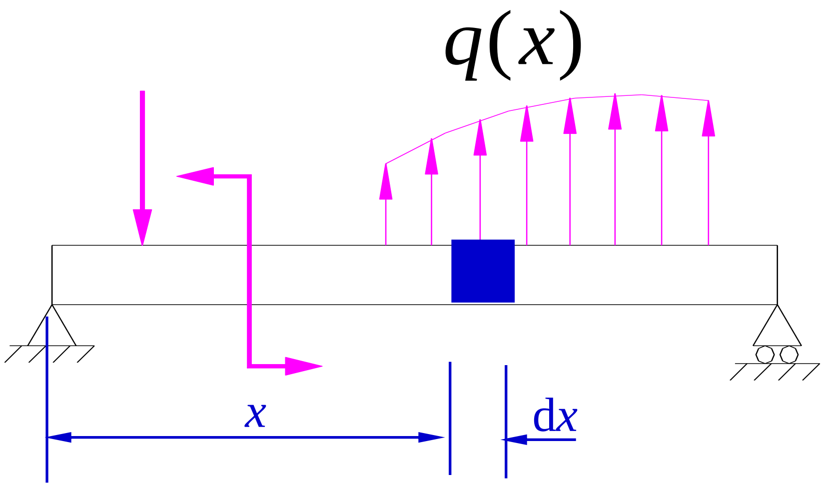
$$M(x) = \frac{ql}{2}x - qx\frac{x}{2}$$

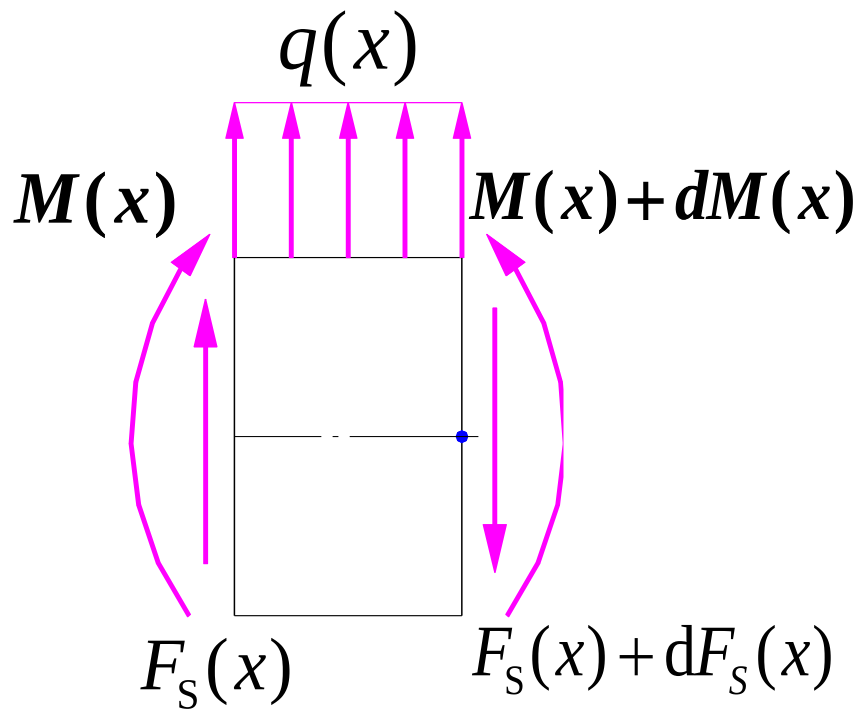
$$= -\frac{q}{2}\left(x - \frac{l}{2}\right)^2 + \frac{ql^2}{8}$$

$$(0 \le x \le l)$$

3. Diagram of internal forces

Relations among Loads, Shearing Forces and Bending Moments





$$F_S(x)+q(x)\mathrm{d}x-F_S(x)-\mathrm{d}F_S(x)=0$$

$$M(x)+\mathrm{d}M(x)-\frac{1}{2}q(x)\mathrm{d}^2x-M(x)-F_S(x)dx=0$$

Relations among Loads, Shearing Forces and Bending Moments

$$F_S(x) + q(x)\mathrm{d}x - F_S(x) - \mathrm{d}F_S(x) = 0$$

$$\Rightarrow \quad q(x) = \frac{dF_S(x)}{dx}, \qquad F_{S2} - F_{S1} = \int_{x_1}^{x_2} q(x)\, dx$$

$$M(x) + \mathrm{d}M(x) - \frac{1}{2} q(x)\mathrm{d}^2 x - M(x) - F_S(x)dx = 0$$

$$\Rightarrow \quad F_S(x) = \frac{\mathrm{d}M(x)}{\mathrm{d}x}, \qquad M_2 - M_1 = \int_{x_1}^{x_2} F_S(x)dx$$

$$\Rightarrow \quad q(x) = \frac{\mathrm{d}M^2(x)}{\mathrm{d}x^2}$$

Sign convention of $q(x)$: up positive / down negative.

Relations among Loads, Shearing Forces and Bending Moments

- Without distributed load: $q(x) = 0$

$$q(x) = \frac{dF_S(x)}{dx} = 0 \quad \Rightarrow \quad F_S(x) = \text{const}$$

Note: the slope of the diagram of shearing forces is zero.

$$\frac{dM(x)}{dx} = F_S(x) = const \quad \Rightarrow \quad M(x) = (\text{const}) \times x + (\text{const})$$

Note: the slope of the diagram of bending moments is constant.

Relations among Loads, Shearing Forces and Bending Moments

- With uniformly distributed load: $q(x)$ = const

$$q(x) = \frac{dF_S(x)}{dx} = \text{const} \Rightarrow F_S(x) = (\text{const}) \times x + (\text{const})$$

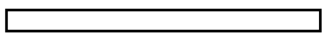
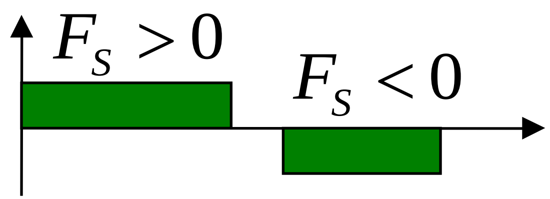
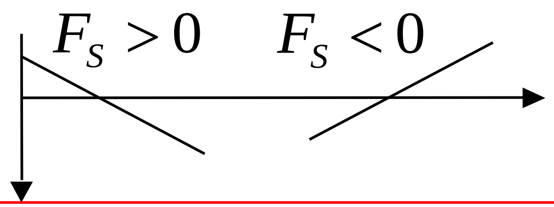
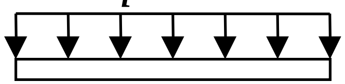
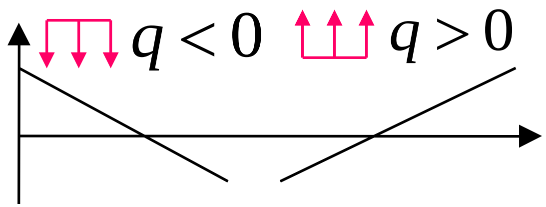
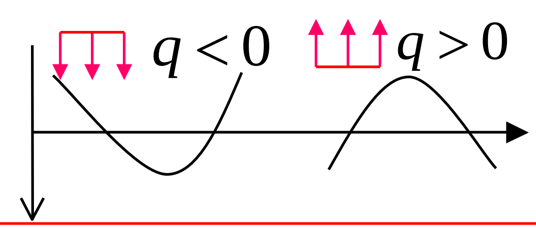
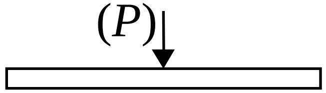
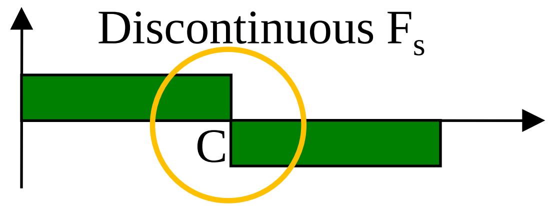
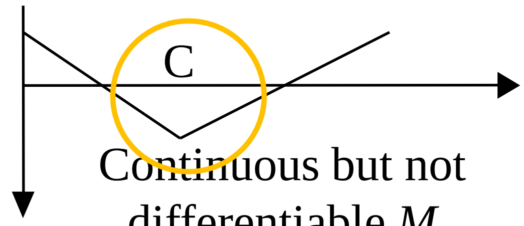
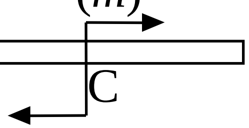
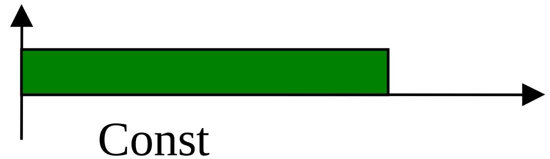
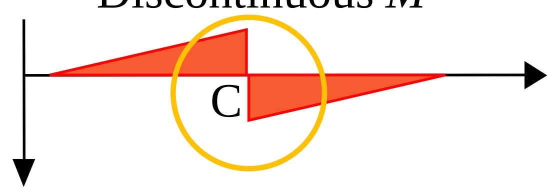
Note: the slope of the diagram of shearing forces is constant.

$$\frac{dM(x)}{dx} = F_S(x) \Rightarrow$$

$$M(x) = (\text{const}) \times x^2 + (\text{const}) \times x + (\text{const})$$

Note: $M(x)$ is a second order polynomial of x.

Relations among Loads, Shearing Forces and Bending Moments

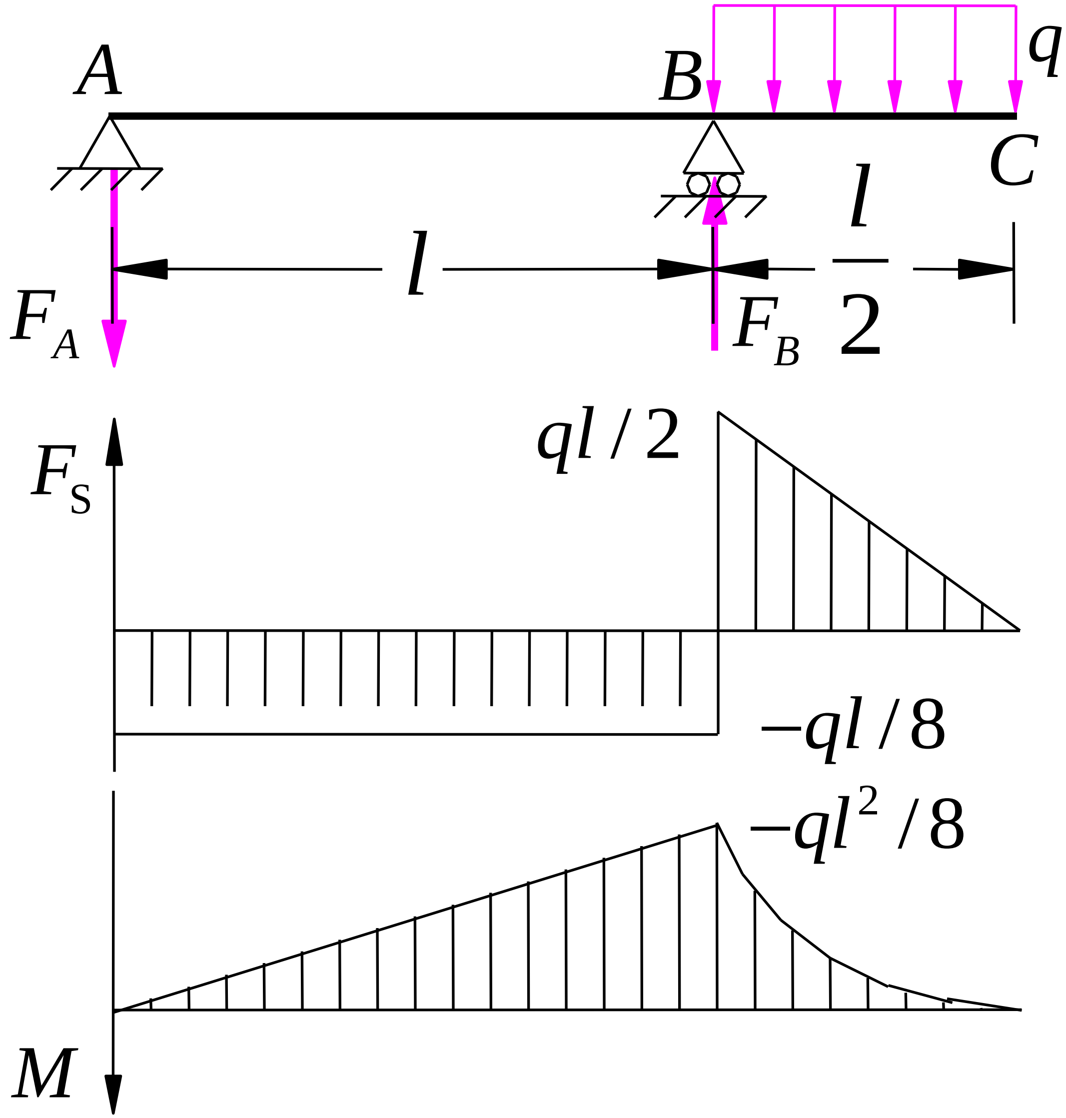
Loads	Diagram of shearing forces	Diagram of bending moments
No load (P=0 & q=0) 		
Uniformly distributed q=const 		
Concentrated load (P) 	Discontinuous F_s 	 Continuous but not differentiable M
Concentrated moment (m) 	 Const	Discontinuous M 

Sample Problem

- Draw the diagram of shearing forces and bending moments.
- Solution:

$$F_A = \frac{ql}{8}$$

$$F_B = \frac{5ql}{8}$$

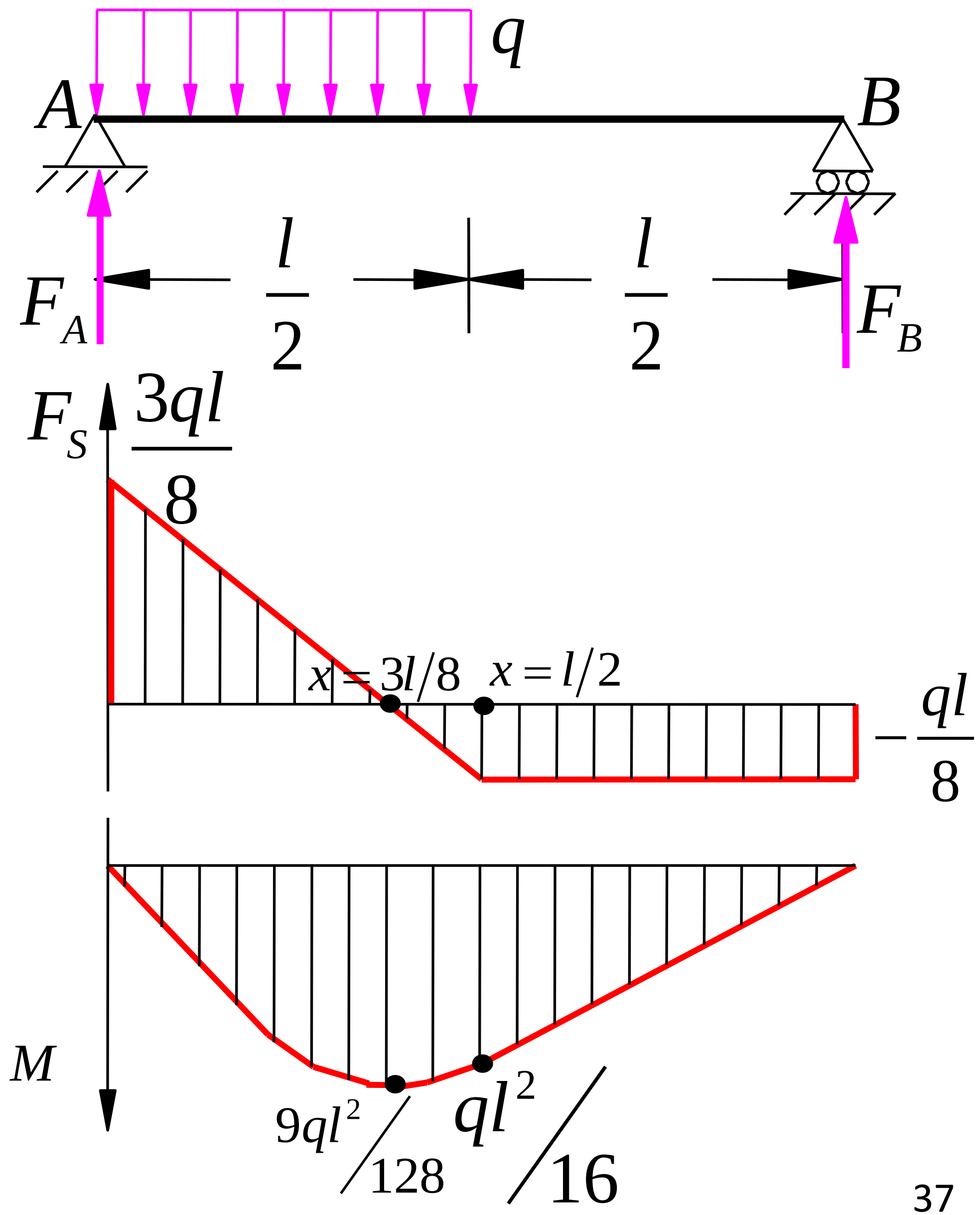


Sample Problem

- Draw the diagram of shearing forces and bending moments.
- Solution:

$$F_A = \frac{3ql}{8}$$

$$F_B = \frac{ql}{8}$$



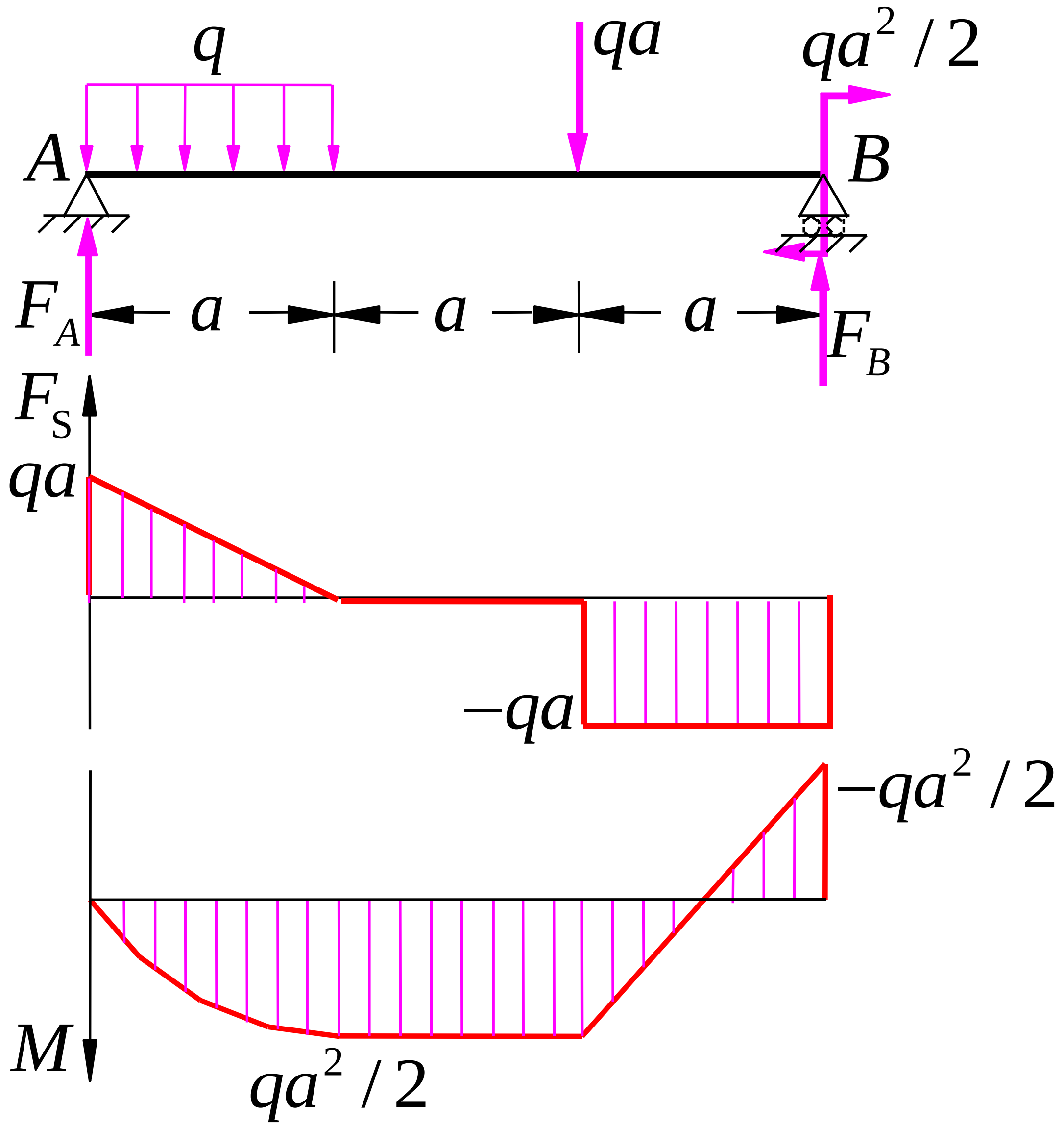
Sample Problem

- Draw the diagram of shearing forces and bending moments.

- Solution:

$$F_A = qa$$

$$F_B = qa$$



Sample Problem

- Draw the diagram of shearing forces and bending moments.

- Solution:

$$F_A = {qa}/{2}$$

$$F_B = {qa}/{2}$$

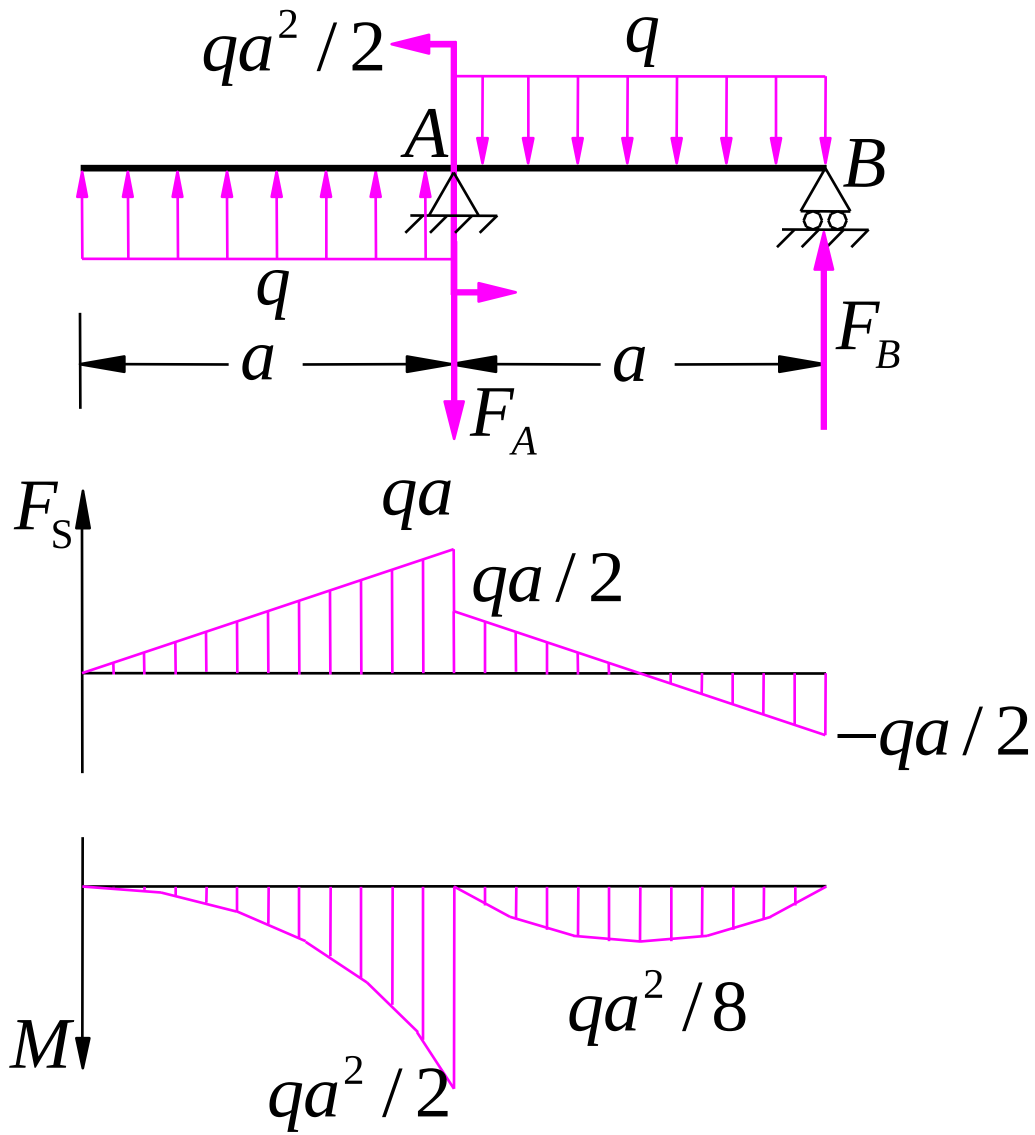


Diagram of Internal Forces for Plane Frames

- Plane frames are composed of prismatic bars with different orientations in the same plane.

- Neighboring bars are connected via rigid joints

- Internal forces acting on cross-sections of component bars include axial force, shearing force, and bending moment.

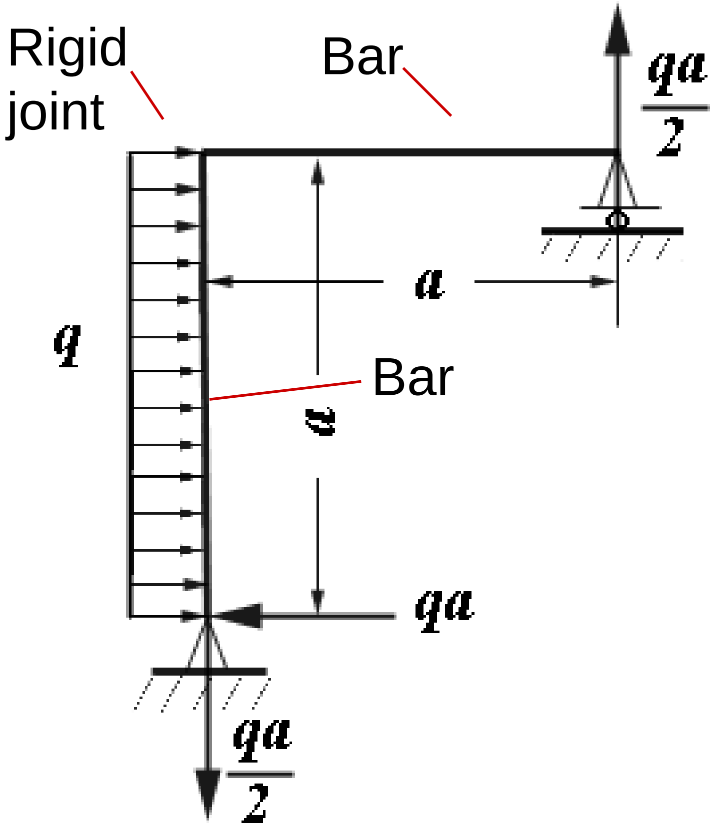
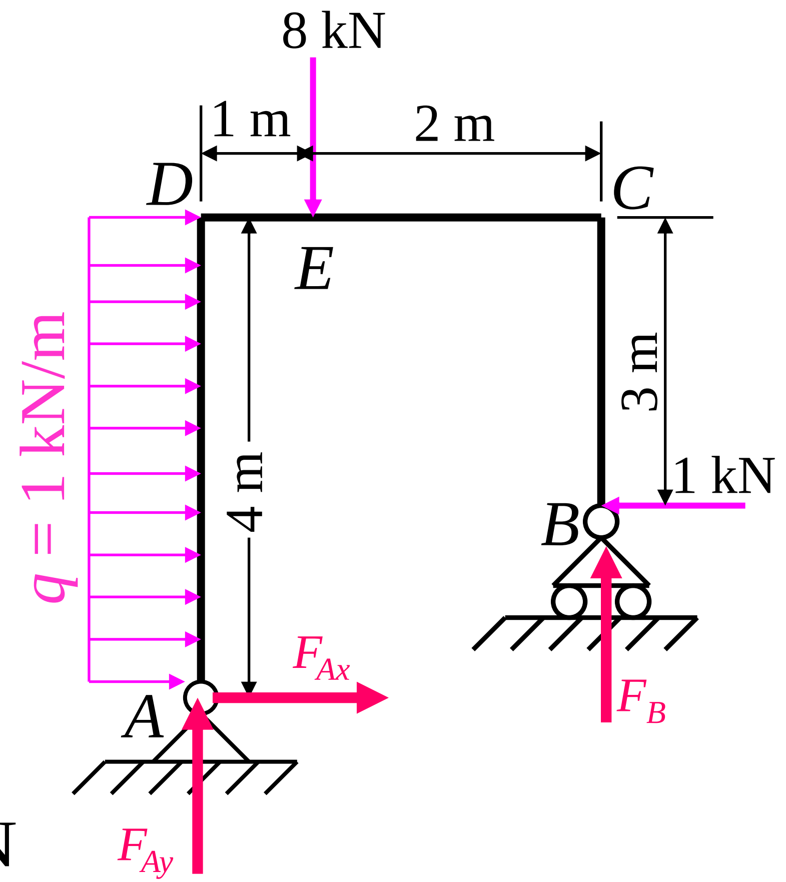


Diagram of Internal Forces for Plane Frames

- Sign convention
  - Axial forces: tension positive / compression negative
  - Shearing forces and bending moments: the same sign convention when the plane frame is observed from inside.
- Drawing convention
  - Axial forces and shearing forces: draw at either side of the bar with sign labeled.
  - Bending moments: draw at the side under tension without sign labeled.

Sample Problem

- Draw the diagram of axial forces, shearing forces and bending moments for the plane frame shown below.
- Solution:



1. reaction forces at the supports.

$$\sum M_A = 0 \Rightarrow$$

$$F_B = \left(8\times 1 + 1\times 4^2/2 - 1\times 1\right)\Big/ 3 = 5\ \text{kN}$$

$$\sum F_x = 0 \Rightarrow$$

$$1\times 4 + F_{Ax} - 1 = 0 \Rightarrow F_{Ax} = -3\ \text{kN}$$

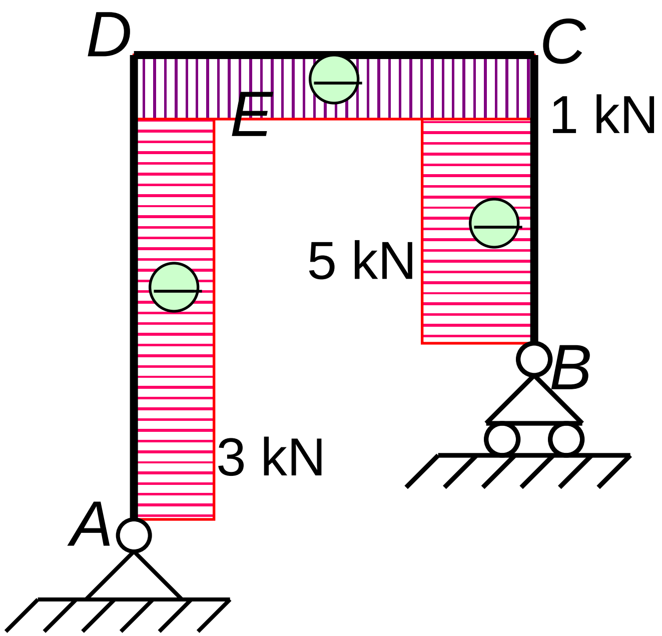
$$\sum F_y = 0 \Rightarrow F_{Ay} + F_B - 8 = 0$$

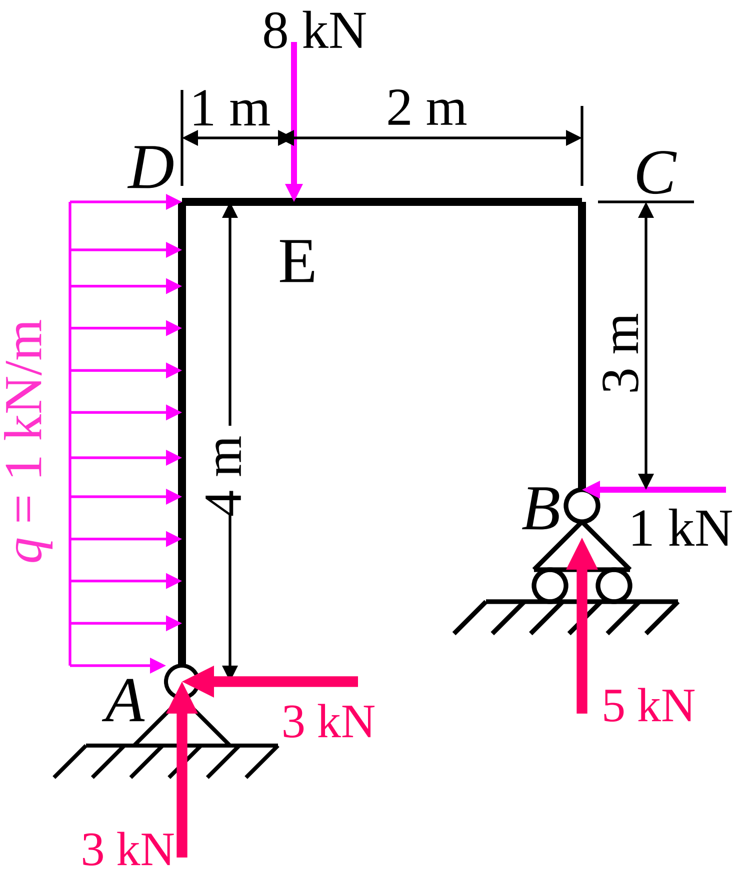
$$\Rightarrow F_{Ay} = 3\ \text{kN}$$

2. Diagram of internal forces

1) Diagram of axial forces

$$BC : F_{N1} = -5\text{ kN}$$
$$DC : F_{N2} = -1\text{ kN}$$
$$AD : F_{N3} = -3\text{ kN}$$



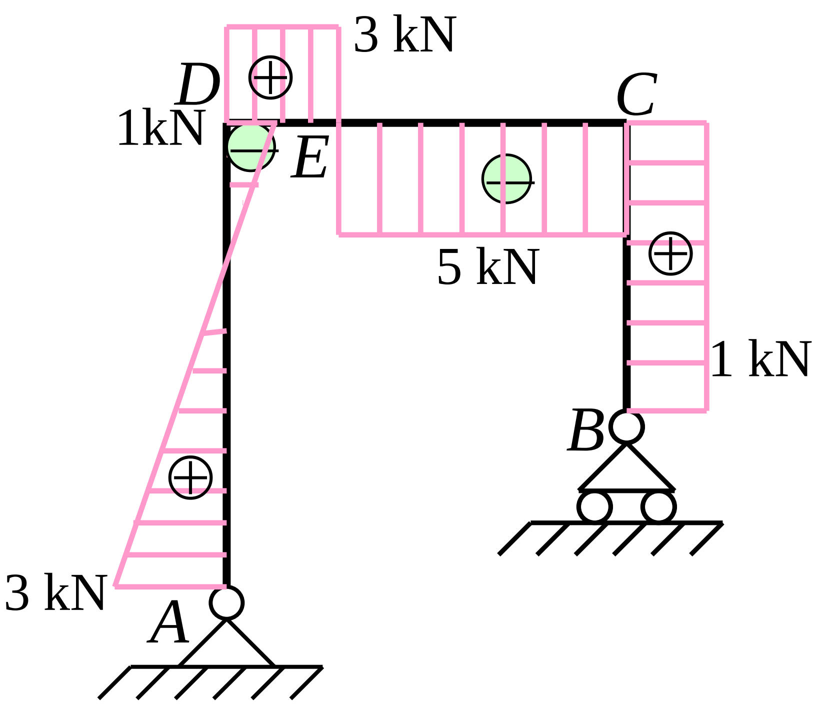


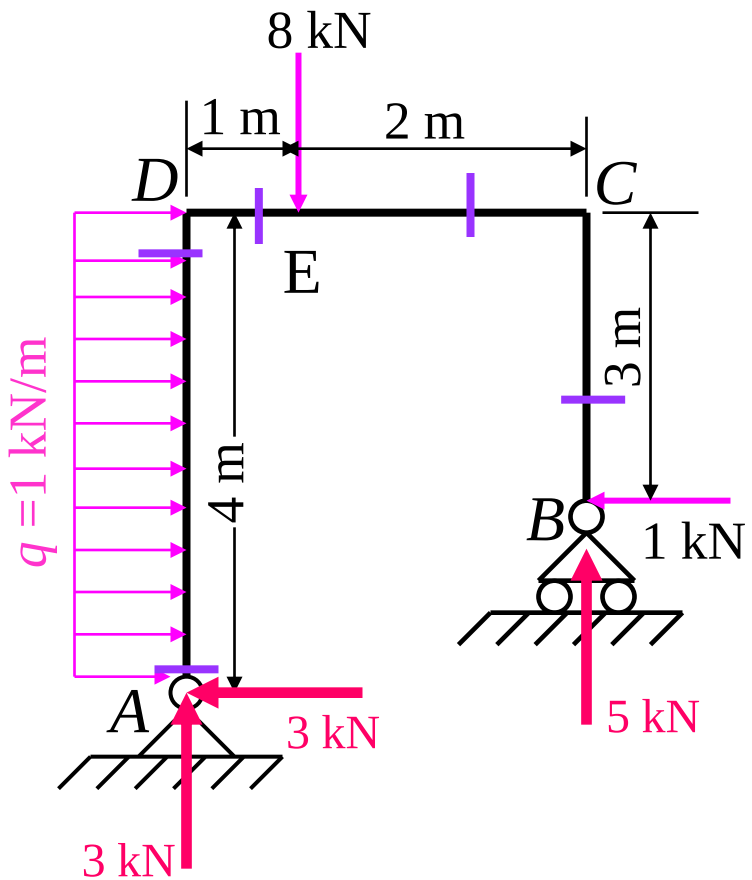
2) Diagram of shearing forces

BC： horizontal line

DC： horizontal line

DA： oblique straight line



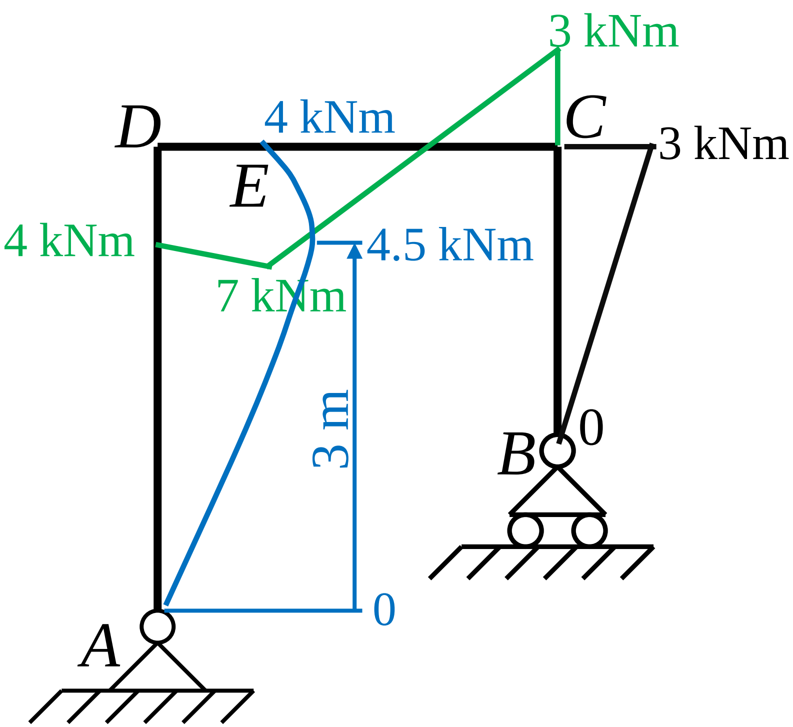


3) Diagram of bending moments

BC: oblique straight line (two-point)

CD: oblique straight line (two-point)

DA: parabola (three-point)



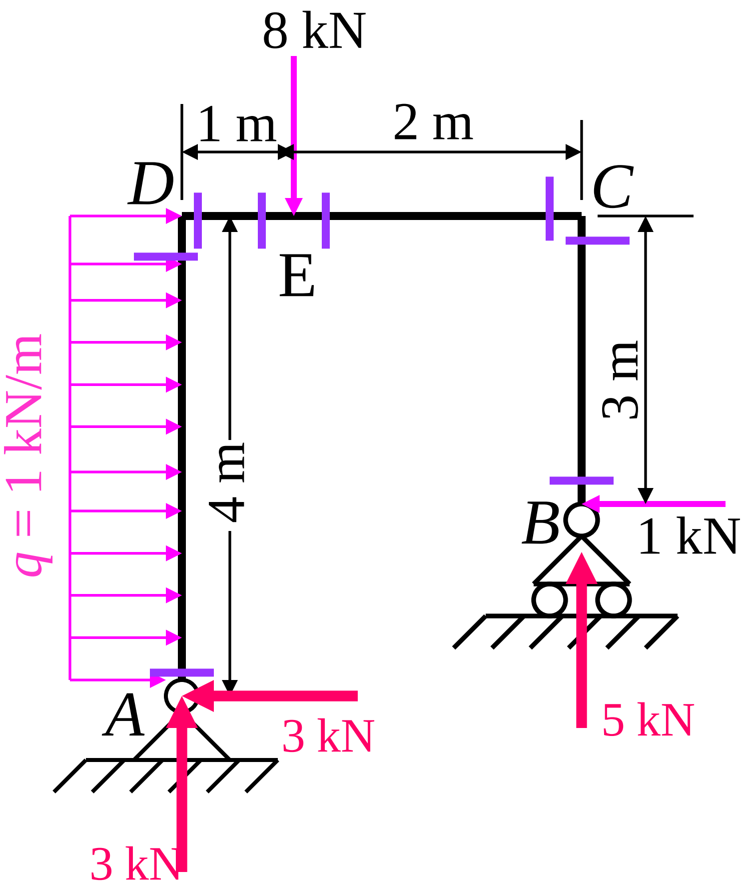
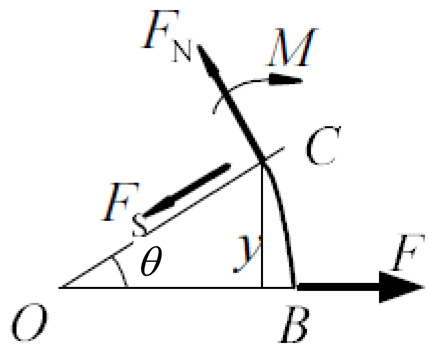
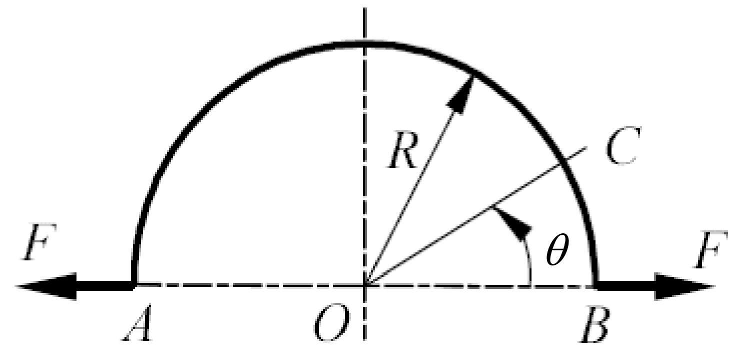


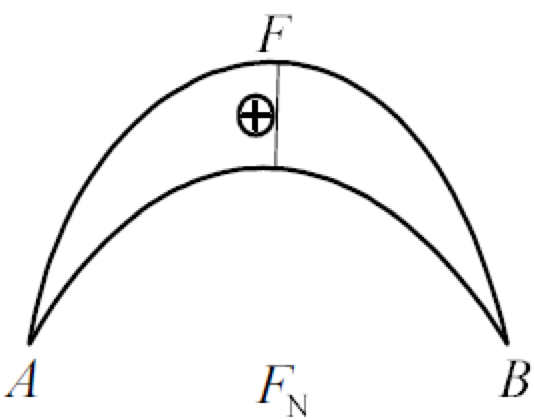
Diagram of Internal Forces for Curved Beams

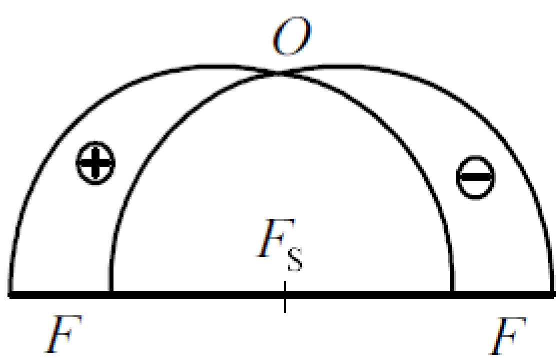
- Draw the diagram of axial forces, shearing forces and bending moments for the curved beam shown below.
- Solution:
- Static equilibrium

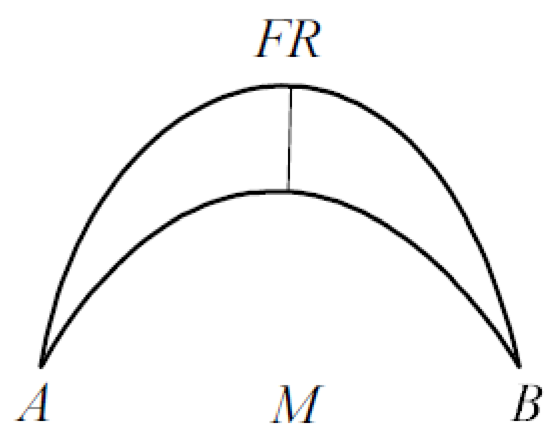




$$F_N(\theta) = F\sin\theta \qquad F_S(\theta) = -F\cos\theta \qquad M(\theta) = Fy = FR\sin\theta$$







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