1. With respect to a basis $\mathbf{e}_i$, a stress tensor is represented by the matrix

$$[\sigma_{ij}] = \begin{bmatrix} -1/10 & 3/5 & 0.0 \\ 3/5 & 6/5 & 0.0 \\ 0.0 & 0.0 & 3/10 \end{bmatrix} \text{MPa}$$

(a) Find the traction vector on the plane $2x_1 - 2x_2 + x_3 = 1$;

(b) Find the magnitude of the traction vector in (a), and the normal and the shear stress on the plane given there;

(c) Find the matrix representing the stress tensor with respect to a basis $\mathbf{e}'_i$, where $\mathbf{e}'_i = \beta_{ij}\mathbf{e}_j$ with

$\boldsymbol{\beta}$ defined by

$$[\beta_{ij}] = \begin{bmatrix} 12/25 & -9/25 & 4/5 \\ 3/5 & 4/5 & 0 \\ -16/25 & 12/25 & 3/5 \end{bmatrix}.$$

2. For the stress tensor given in problem 1, find

(a) The stress invariants I_1, I_2, I_3;

(b) The principal stresses and principal directions;

(c) The octahedral shear stress;

(d) The component matrix of the stress deviator $\hat{\boldsymbol{\sigma}}$ with respect to the basis $\mathbf{e}_i$;

(e) The principal deviatoric invariants J_1, J_2, J_3.

3. Find the maximum shear stress τ_{max} for each stress state given below in MPa.

   (a) Uniaxial tension, $\sigma_1 = 150$;

   (b) Uniaxial compression, $\sigma_2 = -100$;

   (c) Biaxial stress, $\sigma_1 = 50, \sigma_2 = 100$;

   (d) Biaxial stress, $\sigma_1 = 50, \sigma_2 = -50$;

   (e) Triaxial stress, $\sigma_1 = 80, \sigma_2 = \sigma_3 = -40$;

   (f) $\sigma_1 = 50, \sigma_2 = -10, \sigma_3 = 30, \tau_{12} = \tau_{21} = 40$.

4. Show that the principal directions of the deviatoric stress tensor coincide with the principal directions of the stress tensor. Also determine the principal values of the deviatoric stress in terms of principal values of the total stress.

5. (Optional) The plane stress solution for a semi-infinite elastic solid under a concentrated point loading is found to be

$$\sigma_x = -\frac{2Px^2y}{\pi\left(x^2+y^2\right)^2}, \sigma_y = -\frac{2Py^3}{\pi\left(x^2+y^2\right)^2}, \tau_{xy} = -\frac{2Pxy^2}{\pi\left(x^2+y^2\right)^2}$$

Calculate the maximum shear stress at any point in the body and use MATLAB to plot contours of the maximum shear stress. You can compare your results with
the corresponding photoelastic contours shown below.