

---

*S005317*

**Foundations of Solid Mechanics**

*mi@seu.edu.cn*

Changwen Mi

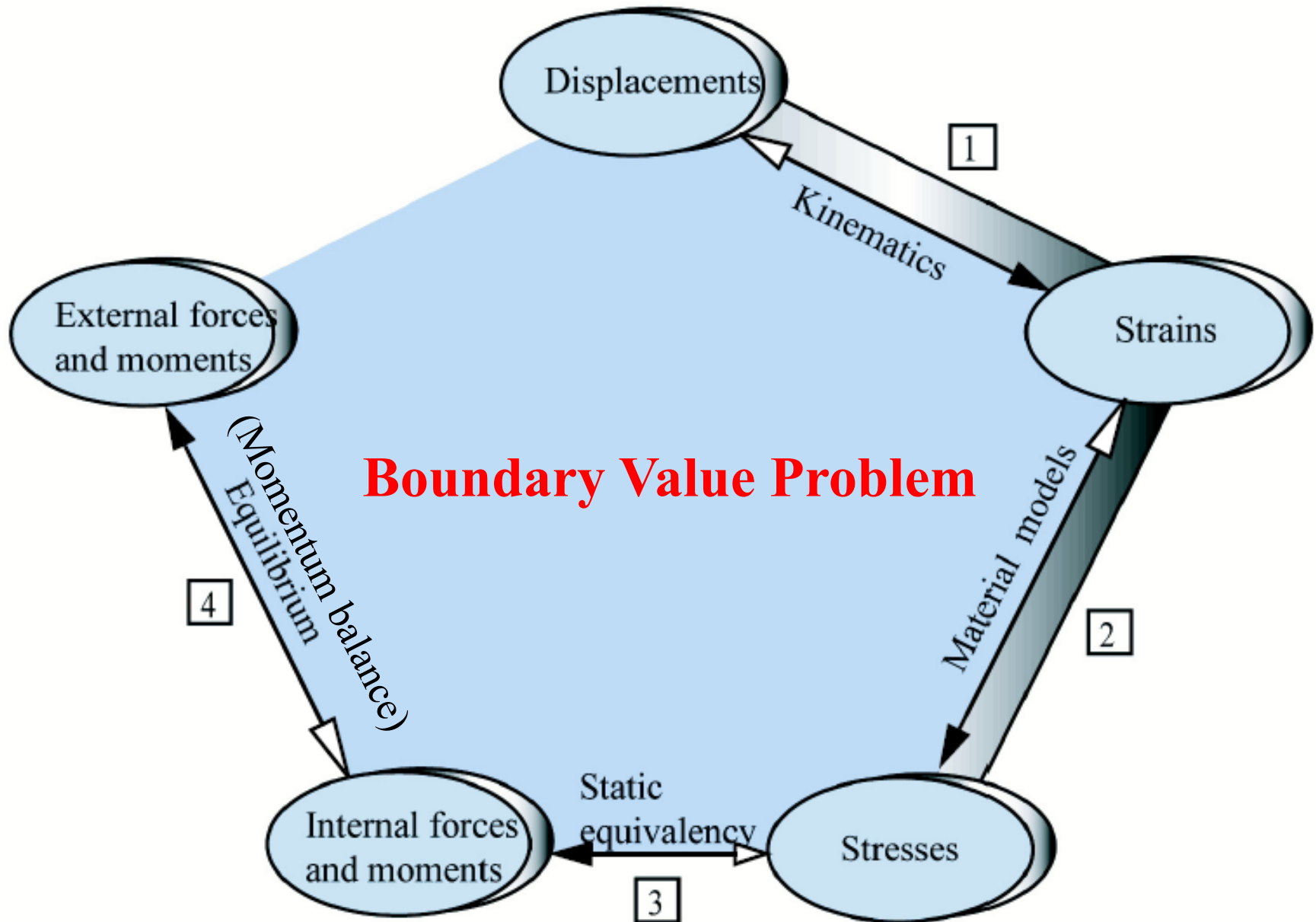
Office: 九龙湖校区土木大楼1401

Phone: 13611568828

# Outline

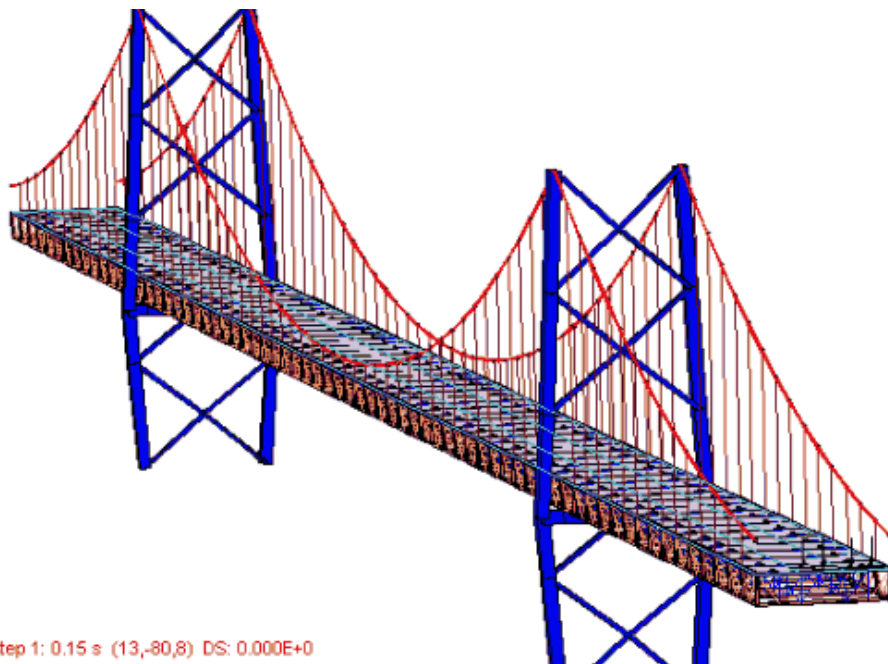
- Scope of the course? (课程涵盖范围)
- Applications (应用)
- Prerequisites (先修知识要求)
- Solution procedure (课程内容概览)
- What did we learn from mechanics of material (材力概念回顾)
- Generalization of concepts (材力概念扩展)
- A 1D example of boundary value problems (1D边值问题示例)
- Course outline (课程主要内容)
- References (参考书目)
- Grading and collaboration policies (评分政策)
- Greek letters (希腊字母)

# Scope of the Course



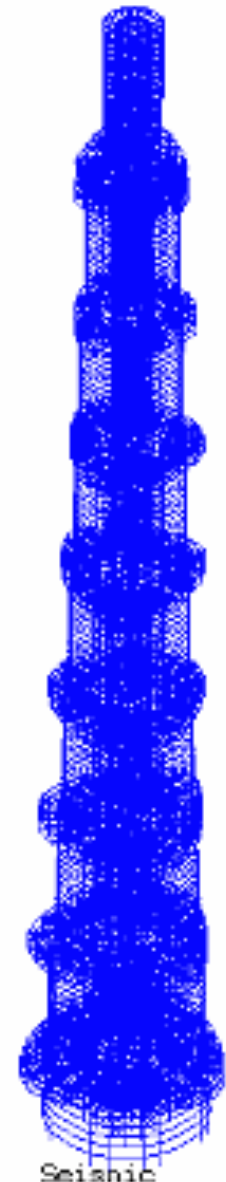
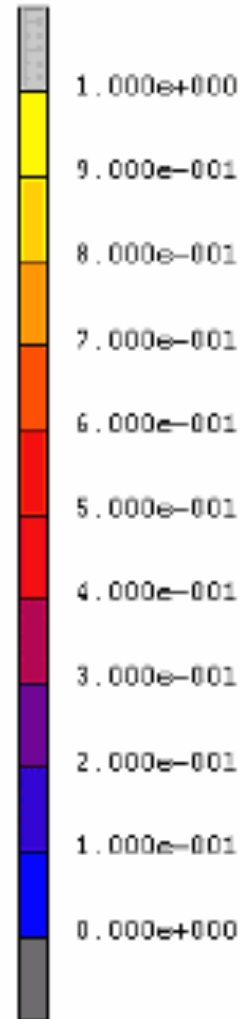
# Applications

- **Geomechanics:** Modeling the shape of planets, tectonics, and earthquake prediction
- **Civil engineering:** Designing structures or soil foundations



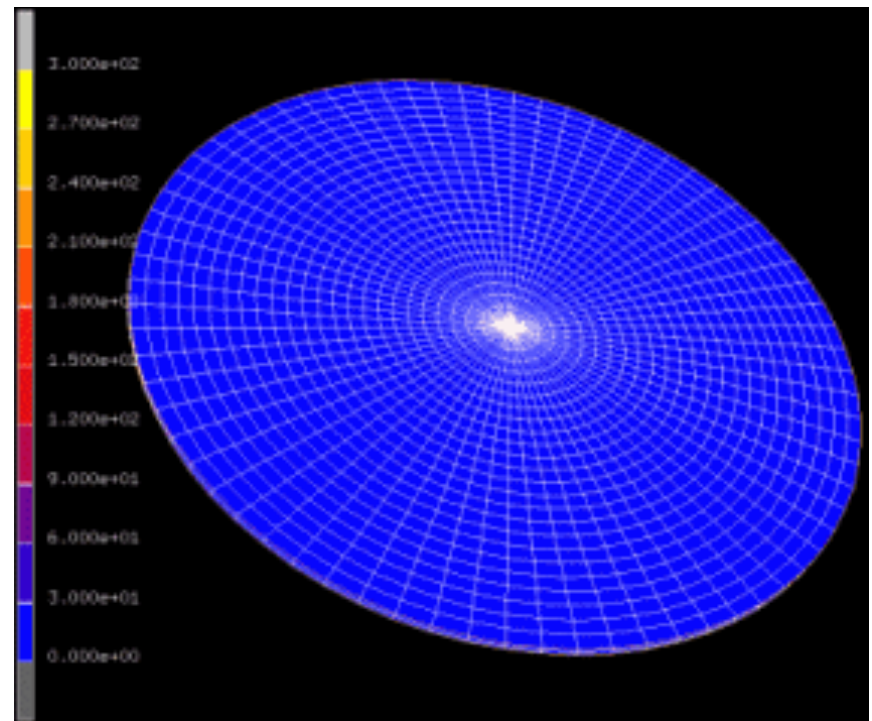
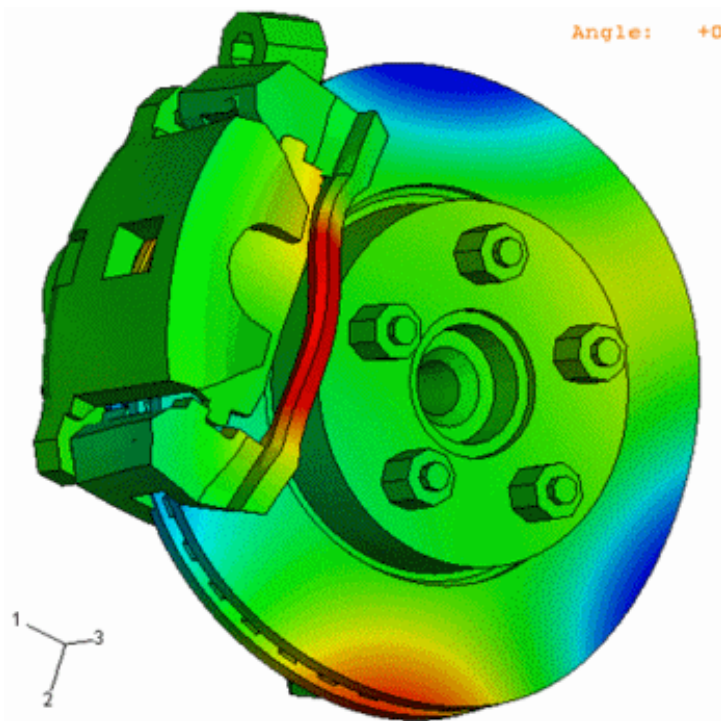
Step 1: 0.15 s (13,-80,8) DS: 0.000E+0

Inc: 380  
Time: 2.843e-001



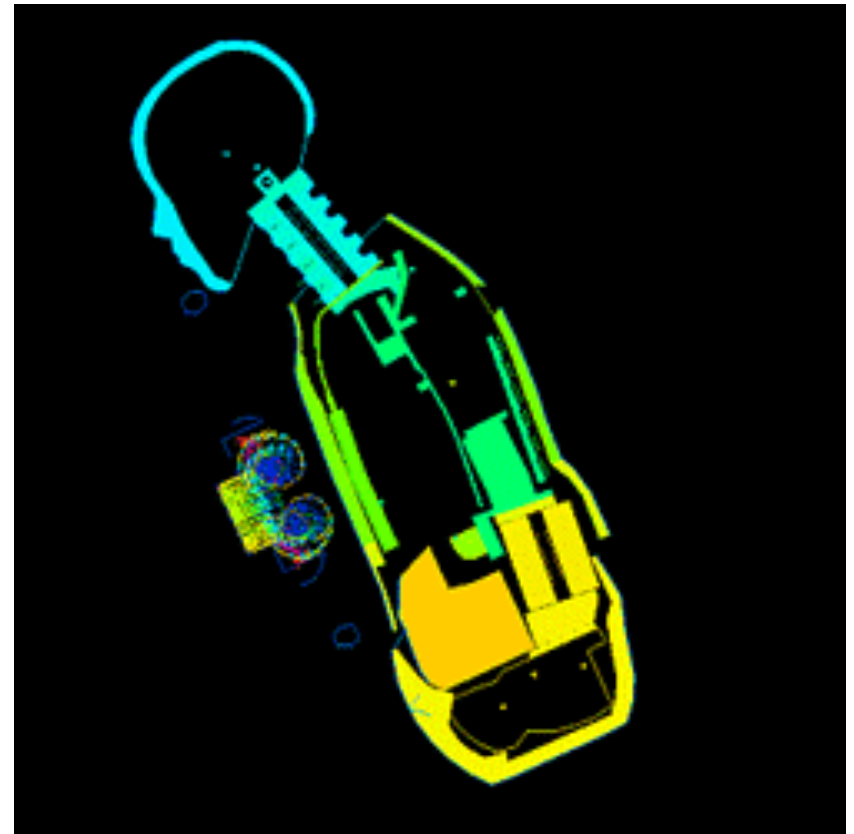
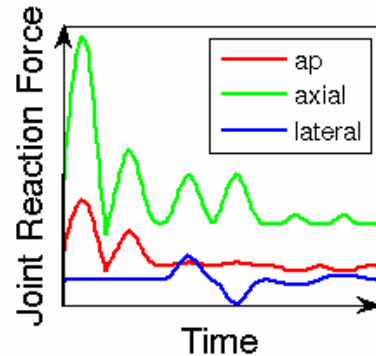
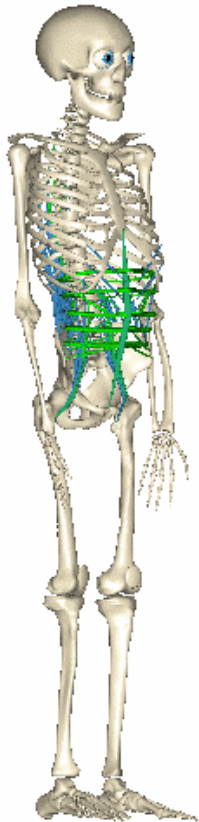
# Applications

- **Mechanical engineering:** Designing load-bearing components for vehicles, engines, or turbines for power generation and transmission, as well as appliances
- **Manufacturing engineering:** Designing processes (such as machining) for forming metals and polymers



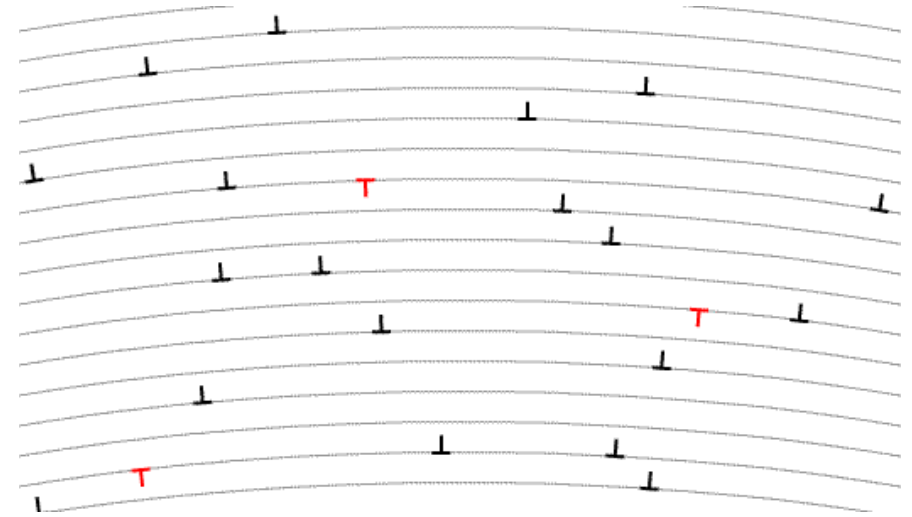
# Applications

- **Biomechanics:** Designing implants and medical devices, as well as modeling stress driven phenomena controlling cellular and molecular processes

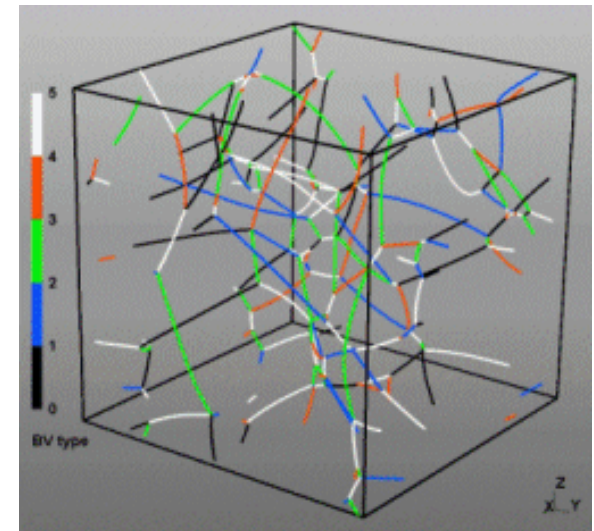
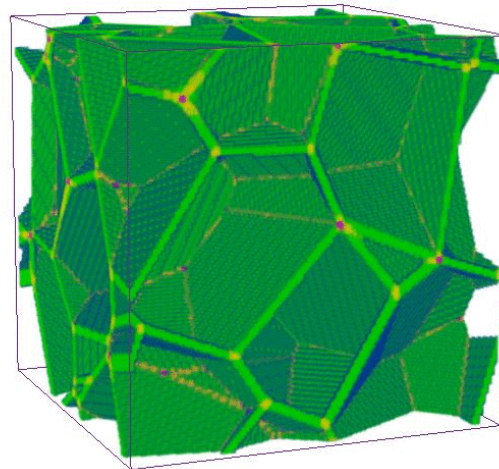
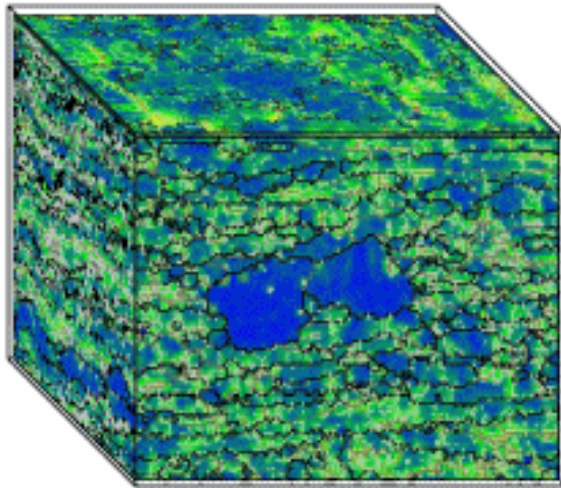


# Applications

- **Materials science:**  
Designing composites,  
alloy microstructures, thin  
films, and developing  
techniques for processing  
materials

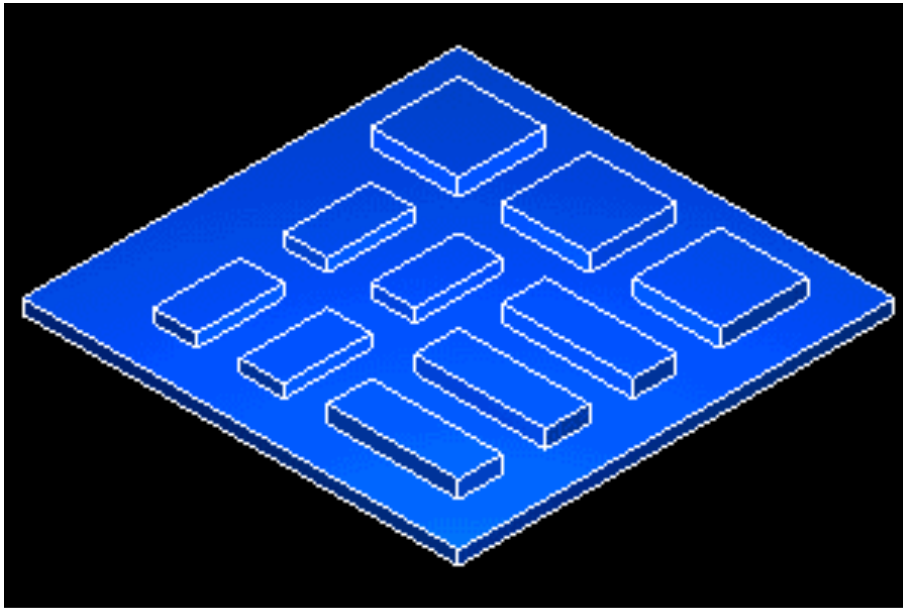


x=166.7 y=224.0 z=64.0

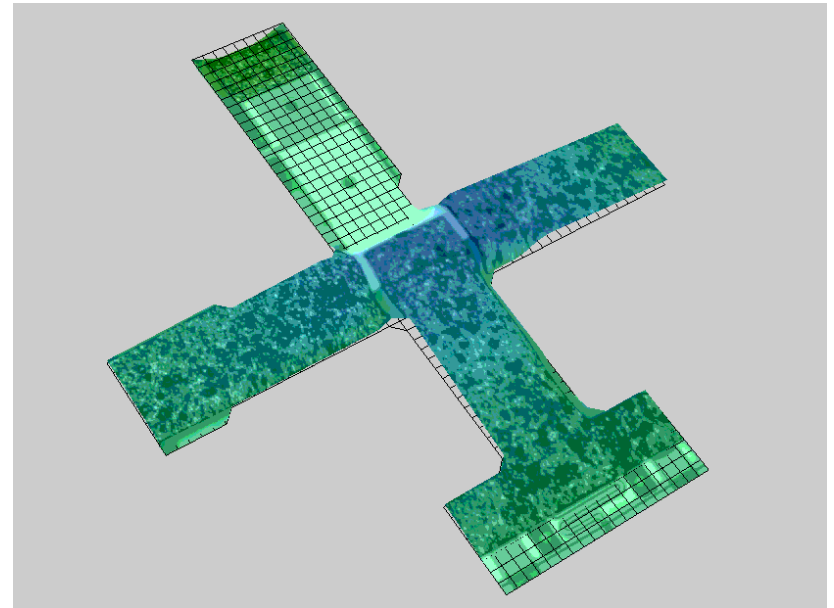


# Applications

- **Microelectronics:** Designing failure-resistant packaging and interconnects for microelectronic circuits



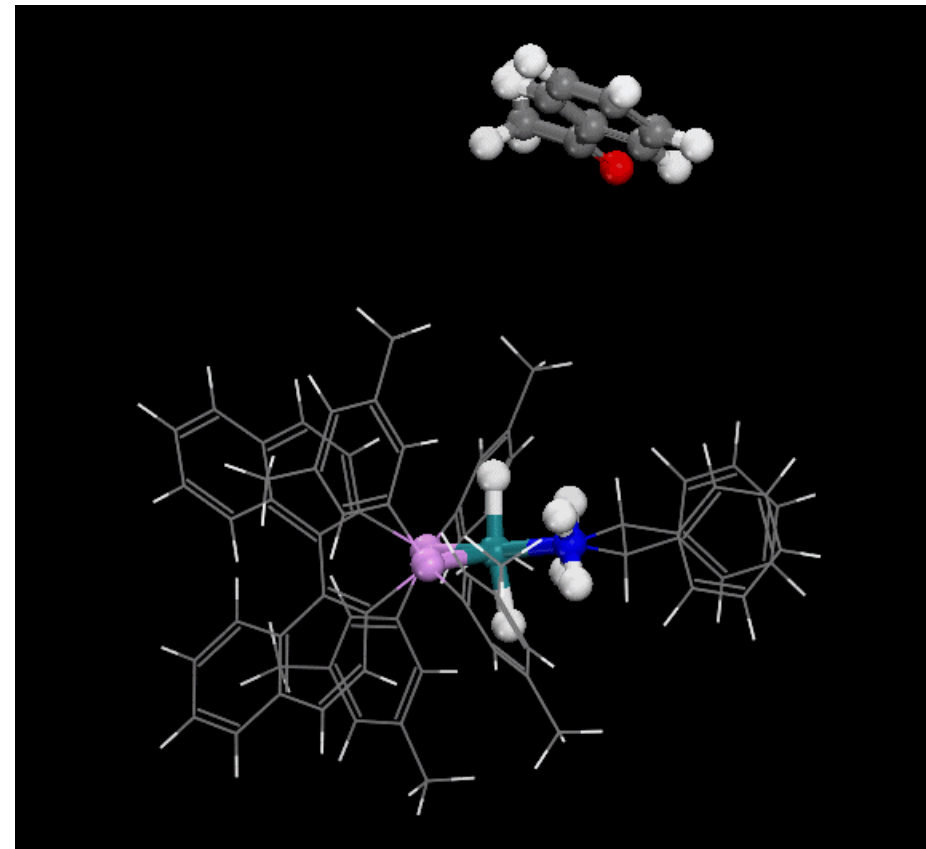
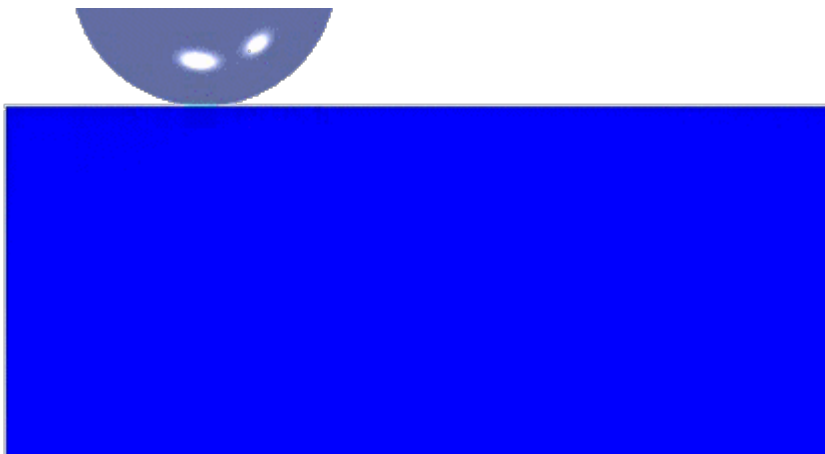
Sample heat analysis





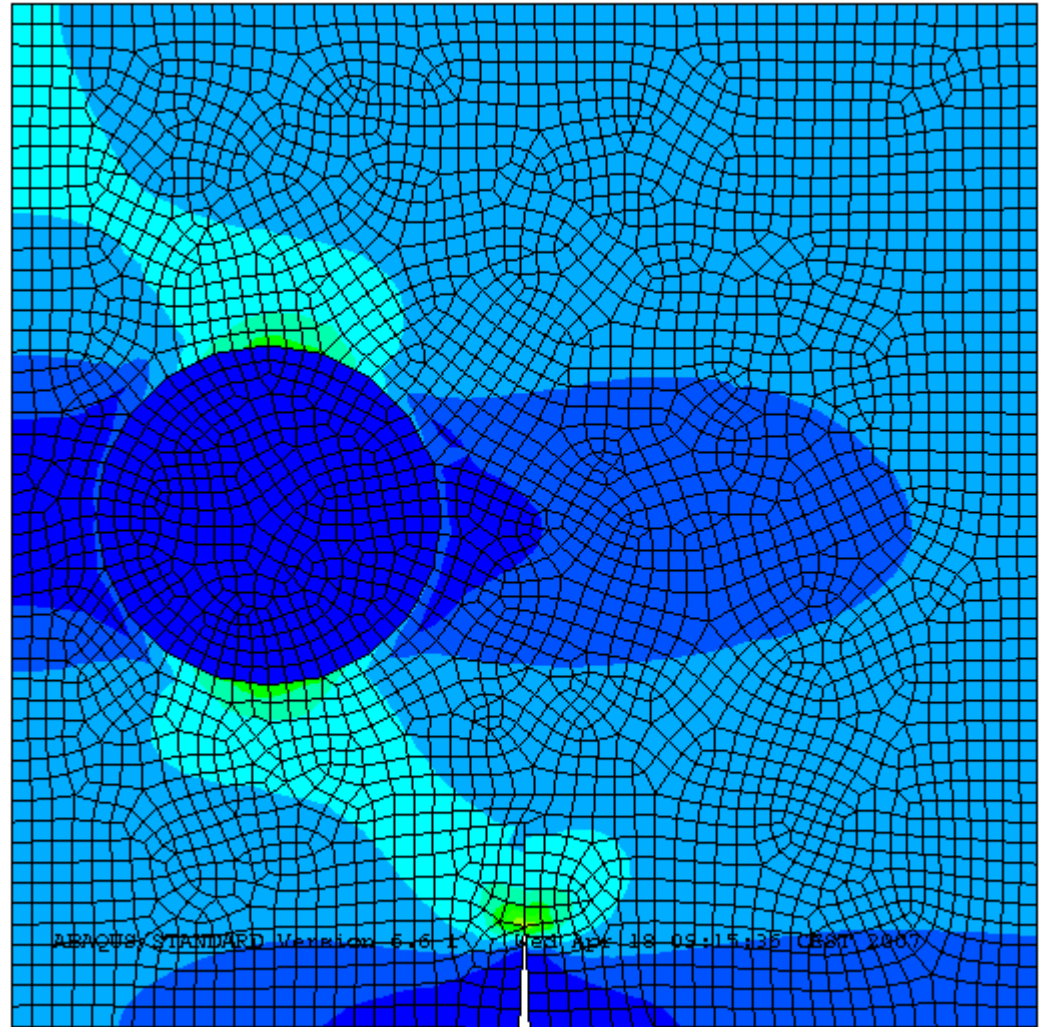
# Applications

- **Nanotechnology:** Modeling stress-driven self-assembly on surfaces, manufacturing processes such as nano-imprinting, and modeling atomic-force microscope/sample interactions



# Applications

- Crack Propagation



# Prerequisites for the Course

- **Mechanics of Materials**
- Mechanical behavior of materials and analysis of stress and deformation in engineering structures and continuous media.
- Topics include concepts of stress and strain; the elastic, plastic, and time dependent response of materials; principles of structural analysis and application to simple bar structures, beam theory, instability and buckling, torsion of shafts; general three-dimensional states of stress; Mohr's circle; stress concentrations.

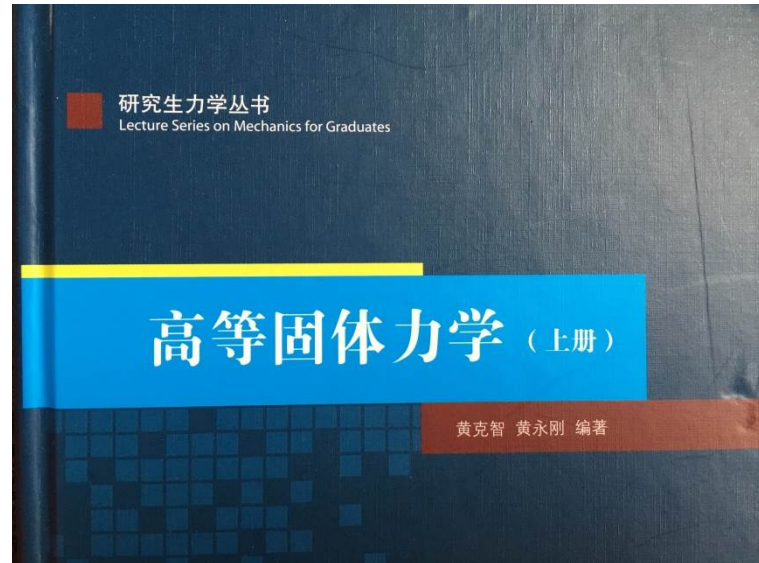
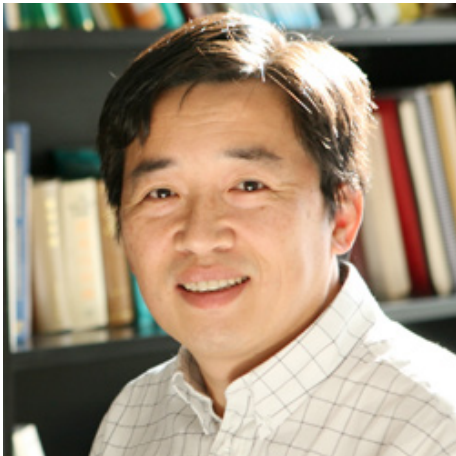
# Prerequisites for the Course

- **Introduction to Engineering**
- An introduction to various engineering disciplines, thought processes, and issues
- Topics include computing in engineering, engineering design, optimization, and estimation.
- Case studies in engineering are used to illustrate engineering fields and scientific principles, including in-depth studies of statics and optics.
- Laboratories and design projects are included.

# Prerequisites for the Course

- **Applied Mathematics**
- Mathematical techniques involving differential equations used in the analysis of physical, biological and economic phenomena
- Emphasis on the use of established methods, rather than rigorous foundations
- First and second order differential equations
- Applications of linear algebra to systems of equations; numerical methods; nonlinear problems and stability; introduction to partial differential equations; ...

# Prerequisites for the Course



## 前 言

2005 年前后，清华大学工程力学系研究生课程“固体本构关系”更名为“高等固体力学”，扩充了应用性的内容。

“高等固体力学”课程主要研究大变形问题，但作为基础，本书上册仍保留了第 1 章“小变形弹性本构关系”，因为这是一个力学工作者必须具备的基础知识。如果不掌握小变形的理论，那么大变形的理论就无从谈起。

研究大变形固体力学，需要两方面的基础：

(1) 张量分析：目前多数教材中用到的张量分析知识还仅限于将张量当作带指标的符号。实际上，张量分析的理论与其用途远比指标符号深刻得多。它不仅可以使推导变得十分简洁，而且还可以清楚地显示出问题本身的物理意义，有时用张量分析方法可以得到一些意想不到的结果。我们可以毫不夸张地说，不懂得张量分析，要阅读和消化现代力学文献是不可能的。清华大学工程力学系每年都为硕士生开设“张量分析”学位课<sup>1)</sup>。

(2) 连续介质力学：包括应力理论、应变理论和本构关系。如果缺少张量分析和连续介质力学的知识，高等固体力学的讲授就不可能达到足够的深度。为此，上册增加了附录：张量分析（介绍）——当然，其中只包含一些最少量的张量分析的必要知识；同时，上册第 2 章“连续介质力学概述”介绍了研究固体力学所必需的连续介质力学基础知识。

# Solution Procedure

---

1. Decide on what to calculate.
2. Identify the geometry of the solid to be modeled.
3. Determine the loading applied to the solid.
4. Decide what physics must be included in the model.
5. Choose (and calibrate) a constitutive law that describes the behavior of the material.
6. Choose a method of analysis.
7. Solve the problem, analytically, numerically or experimentally.

# What to Calculate

---

- The deformed shape of a structure or component subjected to mechanical, thermal, or electrical loading
- The forces required to cause a particular shape change
- The stiffness of a structure or component
- The internal forces (stresses) in a structure or component
- The critical forces that lead to failure by structural instability (buckling)
- Natural frequencies of vibration for a structure or component



# What to Calculate

- Predicting the critical loads to cause fracture in a brittle or ductile solid containing a crack
- Predicting the fatigue life of a component under cyclic loading
- Predicting the rate of growth of a stress-corrosion crack in a component
- Predicting the creep life of a component
- Finding the length of a crack that a component can contain and still withstand fatigue or fracture
- Predicting the wear rate of a surface under contact loading
- Predicting the fretting or contact fatigue life of a surface

# What to Calculate

- Calculating the properties (e.g., elastic modulus, yield stress, stress-strain curve, fracture toughness, etc.) of a composite material in terms of those of its constituents
- Predicting the influence of the microstructure (e.g., texture, grain structure, dispersoids, etc.) on the mechanical properties of metals such as modulus, yield stress, strain hardening, etc.
- Modeling the physics of failure in materials, including fracture, fatigue, plasticity, and wear, and using the models to design failure resistant materials
- Modeling materials processing, including casting and solidification, alloy heat treatments, and thin-film and surface-coating deposition (e.g., by sputtering, vapor deposition, or electroplating)
- Modeling biological phenomena and processes, such as bone growth, cell mobility, cell wall/particle interactions, and bacterial mobility

# Geometry

- Geometry is important for modeling brittle fracture, fatigue failure, or for calculating critical loads required to initiate plastic flow in a component.
- Less important for creep damage, large-scale plastic deformation (e.g., metal forming) or vibration analysis.
- Geometrical features typically only influence local stresses. ( $\leq 3L$ )
- Start with the simplest possible model and gradually refine the calculation.

# Loading

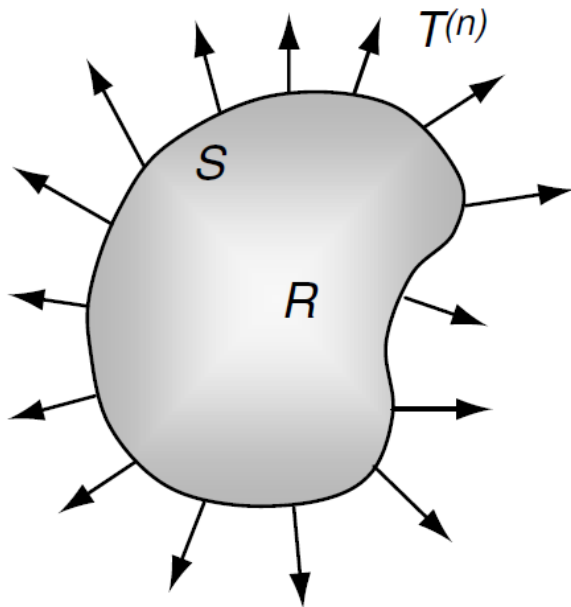
- Displacement boundary conditions  $S_u$
- Traction boundary conditions, either normal or tangential or both,  $S_t$
- Mixed boundary conditions, e.g. horizontal displacements + vertical traction at a point
- Gravitational or electromagnetic body forces
- Contact forces
- Nonuniform thermal expansion
- Materials process such as phase transformation that causes the solid to change its shape

# Difficulties on Applying Loading

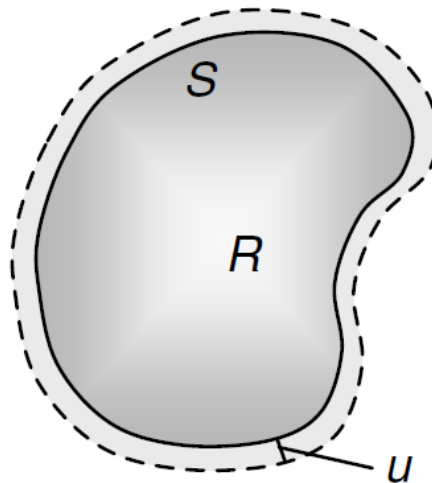
- Can earthquake loading on a building be modeled as a prescribed acceleration of the building's base?
- How to identify pressure loading arising from wind or fluid forces?
- How to estimate frictional coefficients in contact loading?
- How to model attractive forces between two contacting surfaces in nanoscale and biological applications?
- In these cases, standards are helpful...

# Boundary Conditions

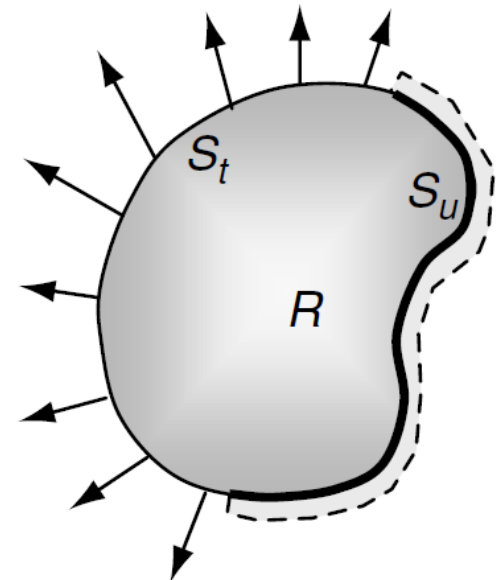
- Must specify exactly three components of either displacement  $(u_1, u_2, u_3)$ , or traction  $(t_1, t_2, t_3)$ , or mixed, e.g.  $(u_1, t_2, t_3)$  or  $(u_1, u_2, t_3)$ , at each point on the boundary.
- Conservation of both linear and angular momentum must be satisfied.



Traction Conditions



Displacement Conditions



Mixed Conditions

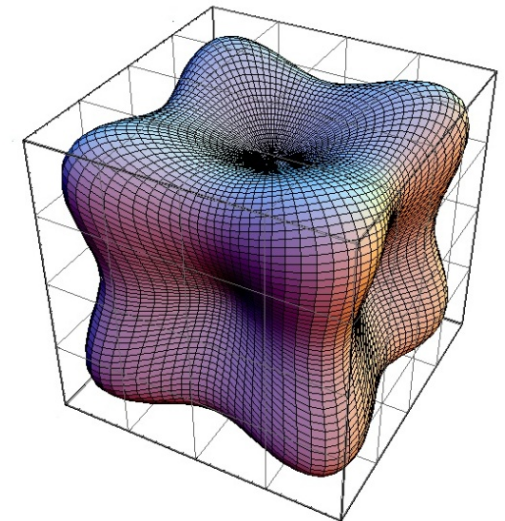
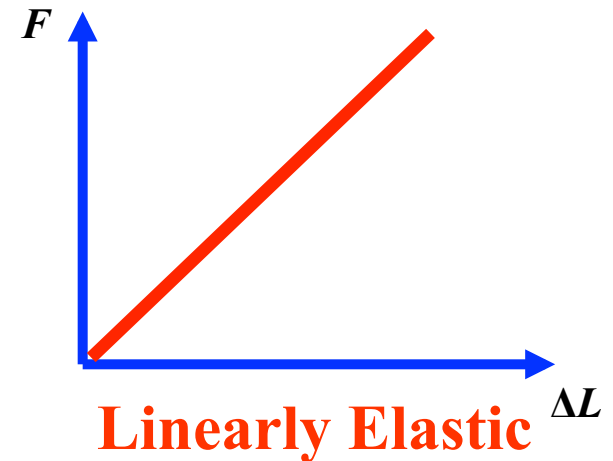
# What Physics to Include

---

- Consider temperature, electric or magnetic fields, or mass/fluid diffusion through the solid?
- Perform a transient heat conduction analysis?
- Do a dynamic analysis or a static analysis?
- Are you solving a coupled fluid/solid interaction problem?

# Define Material Behavior

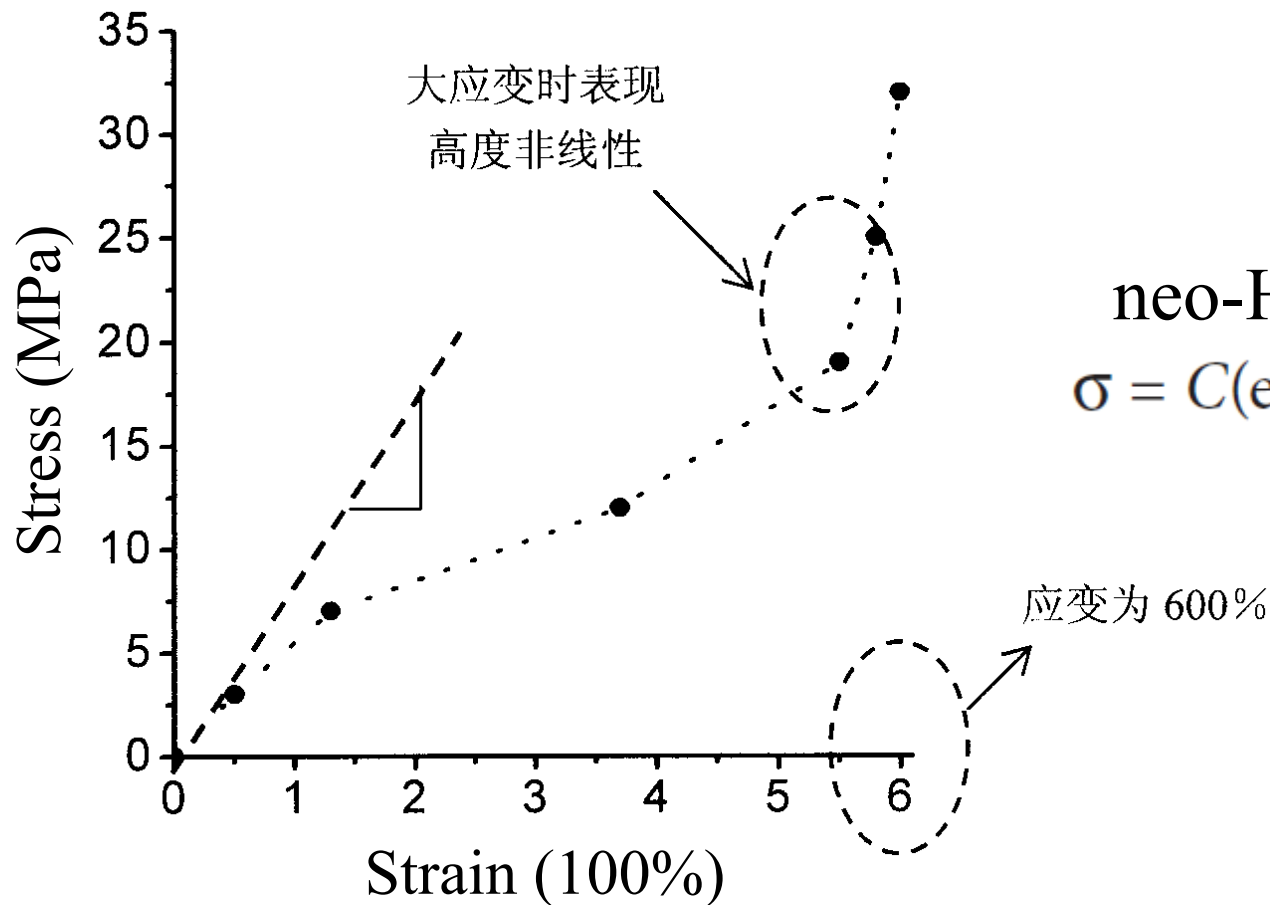
- **Isotropic linear elasticity:** for polycrystalline metals, ceramics, glasses, and polymers; small strain; only two material constants; readily available; for deflection calculations, fatigue analysis, and vibration analysis
- **Anisotropic linear elasticity:** for reinforced composites, wood, and single crystals of metals and ceramics; stiffer in some directions than others; small strain; 3 to 21 constants; available





# Define Material Behavior

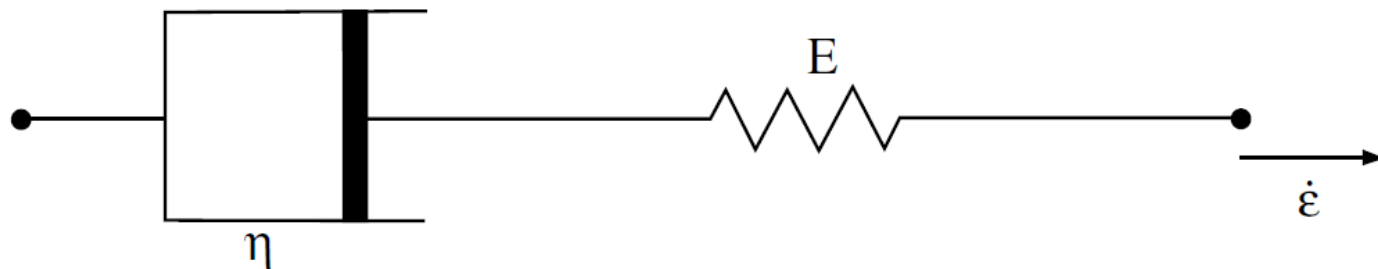
- **Hyper-elasticity:** for rubbers and foams; nearly incompressible ( $\nu \rightarrow 0.5$ ); several material parameters; hard to find; experimental calibration required



neo-Hookean solid  
$$\sigma = C(\exp(2\varepsilon) - 1/\exp(\varepsilon))$$

# Define Material Behavior

- **Viscoelasticity:** for polymeric materials, polymer-based composites, biological tissue, and can also model slow creep in amorphous solids such as glass; at least three parameters; behavior varies widely among materials and is highly temperature dependent; almost certainly requires experimental calibration

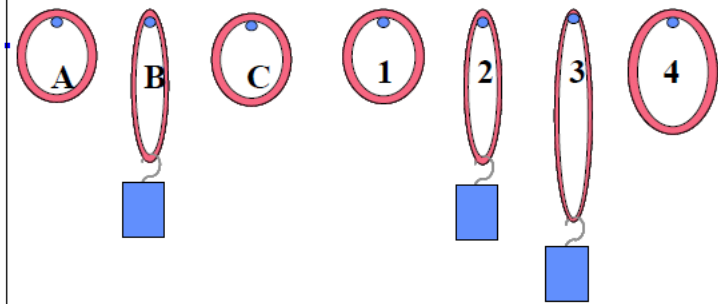
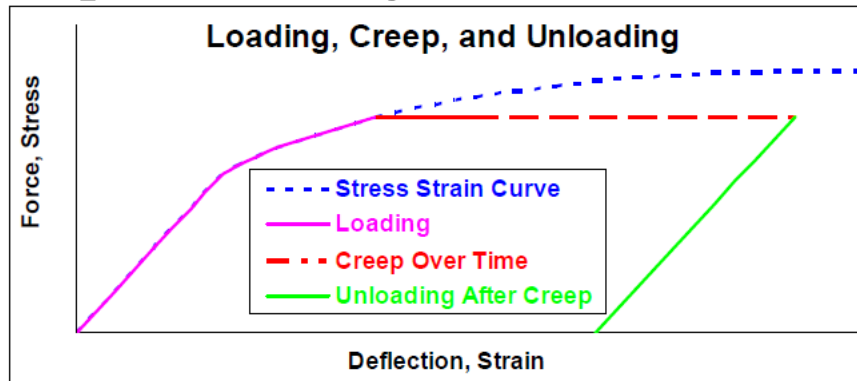


Maxwell Element: spring stiffness  $E$ , dashpot viscosity  $\eta$

- Increase in strain under constant stress:  $\dot{\epsilon} = \dot{\sigma} / E + \eta \sigma$
- Show hysteresis during cyclic loading:  $\dot{\sigma} = E\dot{\epsilon} + \lambda \dot{\epsilon}$

# Define Material Behavior

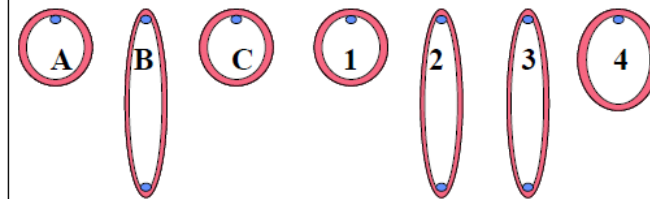
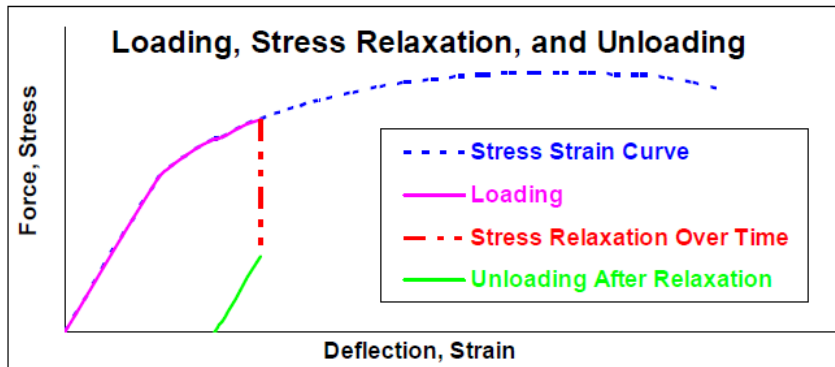
- $\sigma = \varepsilon \cdot f(t)$  - linear visco-elasticity
- $\sigma = f(\varepsilon, t)$  - non-linear visco-elasticity
- Creep: increasing strain with time under constant stress



Stress-Strain Curve with Creep

Creep Example

- Relaxation: decreasing stress with time under constant strain

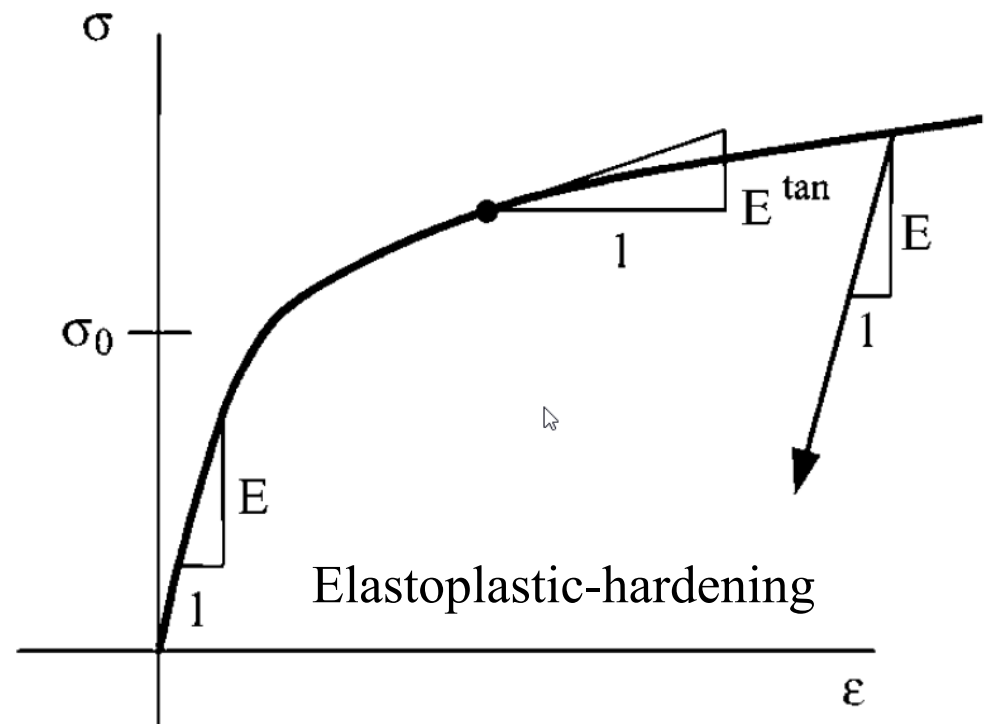
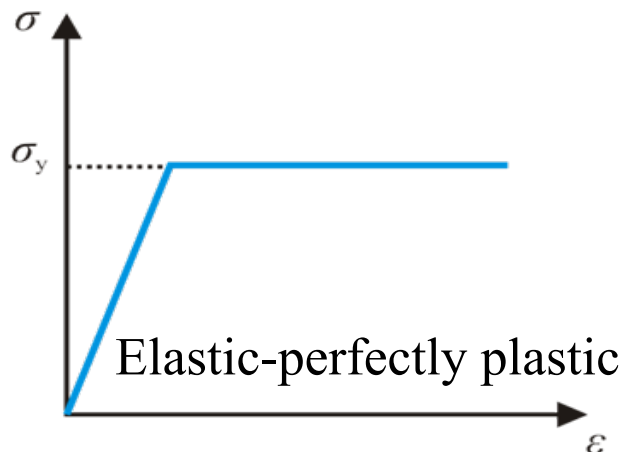
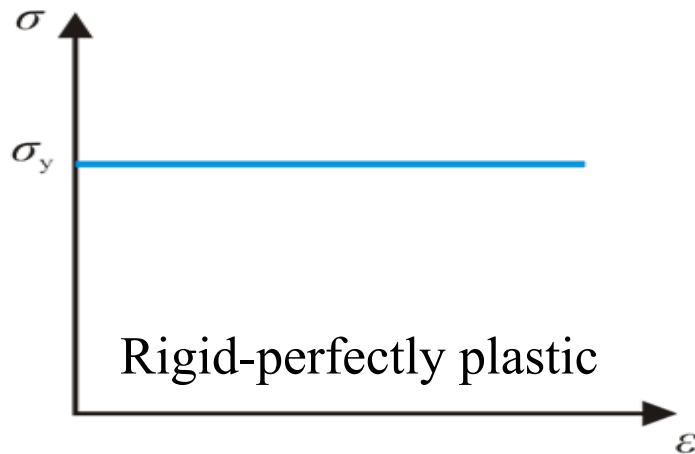


Stress-Strain Curve with Stress Relaxation

Stress Relaxation Example

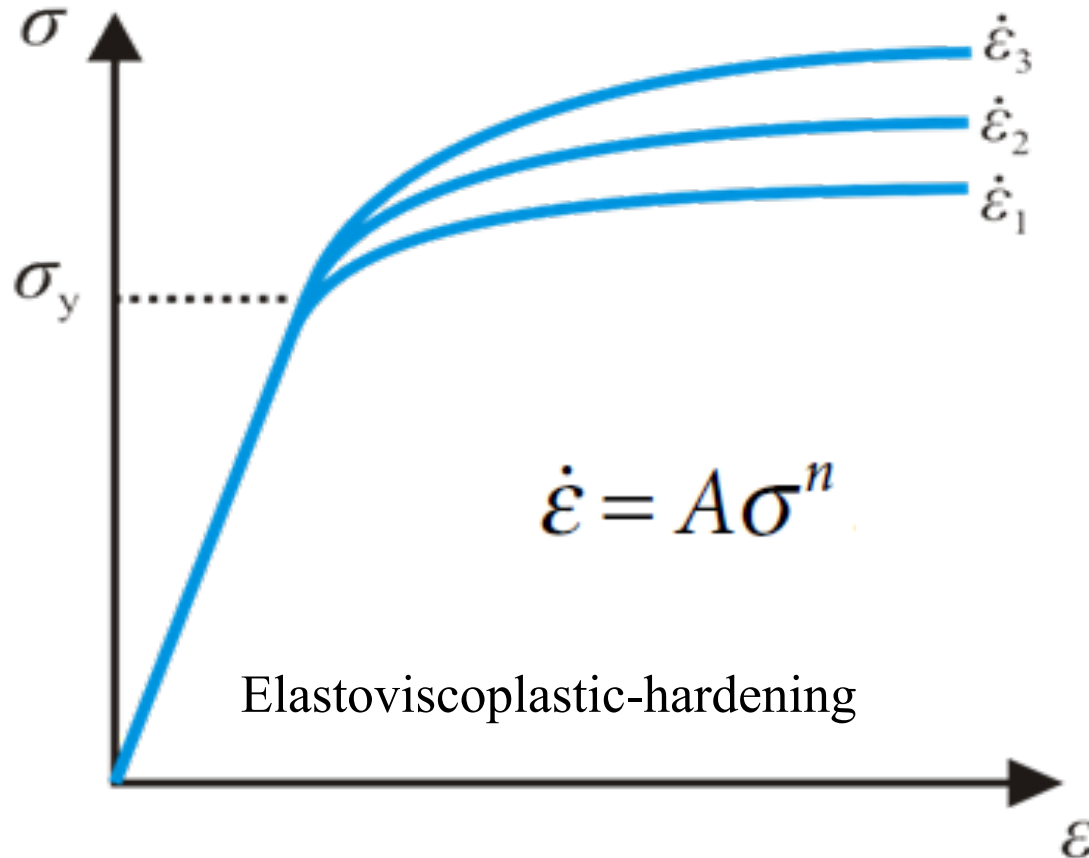
# Define Material Behavior

- **Rate-independent metal plasticity:** for calculating permanent deformation; strain hardening; Gurson plasticity model for void growth; require calibration



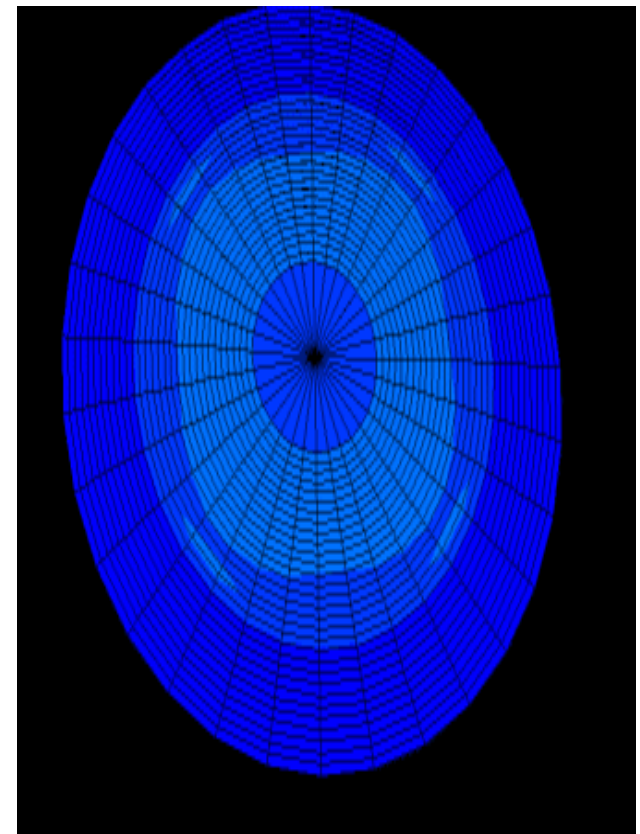
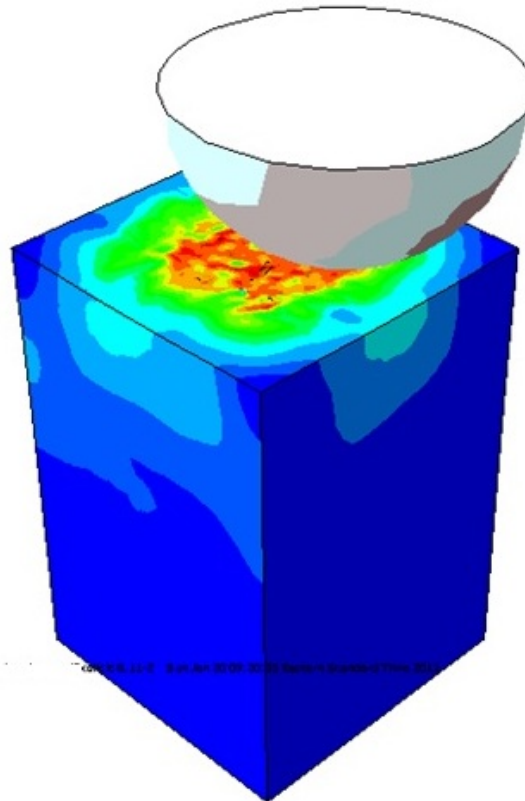
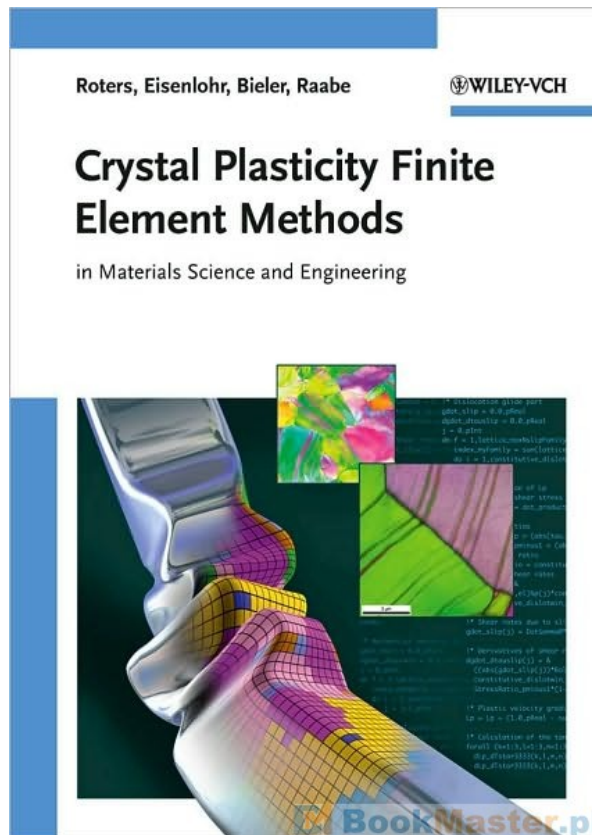
# Define Material Behavior

- **Elastoviscoplasticity:** the flow stress of metal increase with strain rate; for high-speed machining , explosive shock loading, and creep; complex models require calibration



# Define Material Behavior

- **Crystal plasticity:** for anisotropic plastic flow in a single crystal of a metal; for metal-forming processes; still under development because material data are not easily found and are laborious and expensive to measure



# Define Material Behavior

- **Strain gradient plasticity:** to model the behavior of very small volumes of a metal (less than 100  $\mu\text{m}$ ) which are typically stronger than bulk samples; still under development
- **Discrete dislocation plasticity:** models plastic flow in very small volumes of material by tracking the nucleation, motion, and annihilation of individual dislocations; difficult to calibrate.
- **Critical state plasticity (cam-clay)**
- **Pressure-dependent viscoplasticity**
- **Atomistic models**

# Method: Analytical Solutions

- **Exact solutions (closed-form)**
- 2D linear elastic solids, under static or dynamic loading by integral transforms, stress function methods, and complex variable methods
- 2D viscoelastic solids
- Certain simple 3D linear elastic problems using integral transforms
- 2D (plane strain) deformation of rigid plastic solids using slip line fields
- **Semi-analytical solutions:** infinite power series & improper integral representations, followed by numerical evaluation



# Method: Analytical Solutions

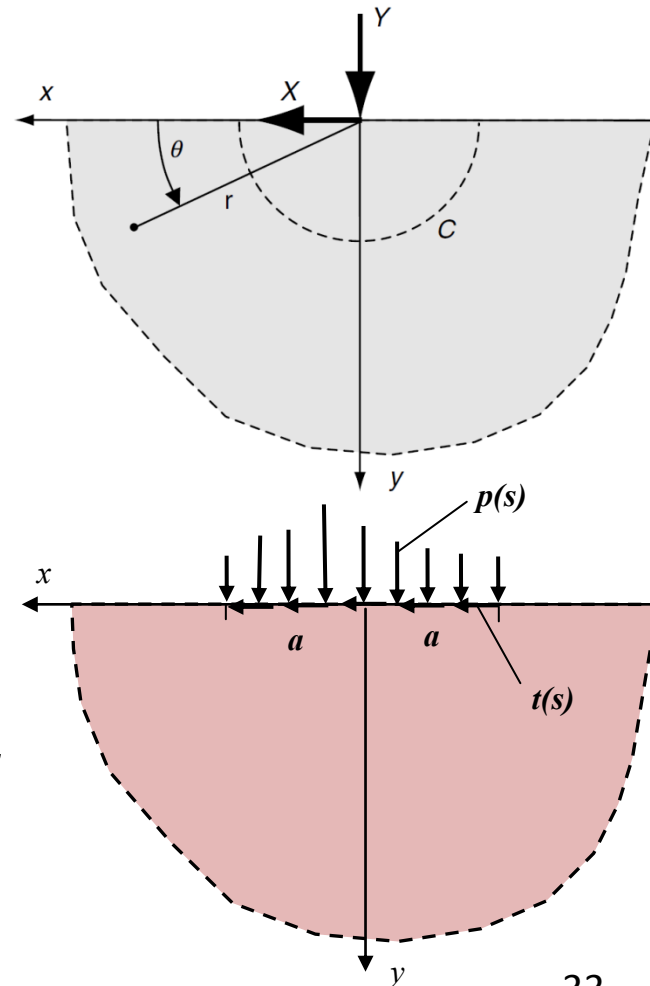
- **The principle of superposition** can be used to extend the point force solution to arbitrary pressures acting on a surface.

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} = -\frac{2}{\pi} \frac{1}{r^4} \begin{bmatrix} Yx^2y + Xx^3 & Yxy^2 + Xx^2y \\ Yxy^2 + Xx^2y & Yy^3 + Xxy^2 \end{bmatrix}$$

$$\sigma_x = -\frac{2}{\pi} \int_{-a}^a \frac{p(s)(x-s)^2 y}{[(x-s)^2 + y^2]^2} ds - \frac{2}{\pi} \int_{-a}^a \frac{t(s)(x-s)^3}{[(x-s)^2 + y^2]^2} ds$$

$$\sigma_y = -\frac{2}{\pi} \int_{-a}^a \frac{p(s)y^3}{[(x-s)^2 + y^2]^2} ds - \frac{2}{\pi} \int_{-a}^a \frac{t(s)(x-s)y^2}{[(x-s)^2 + y^2]^2} ds$$

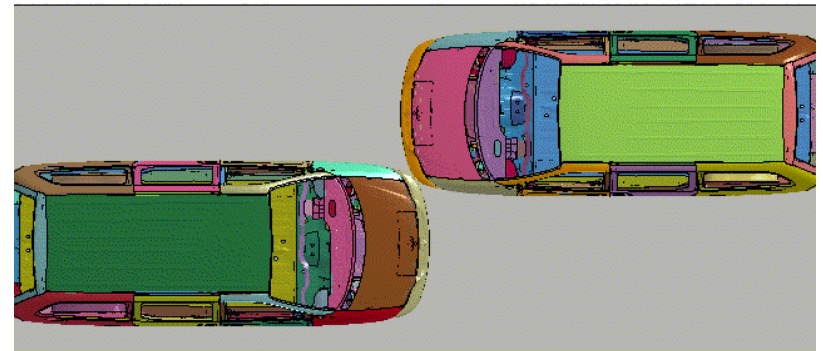
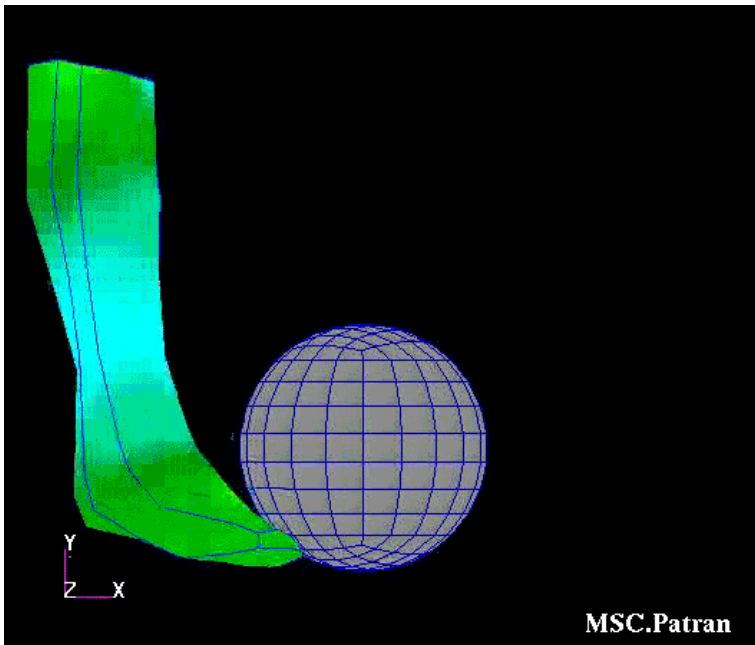
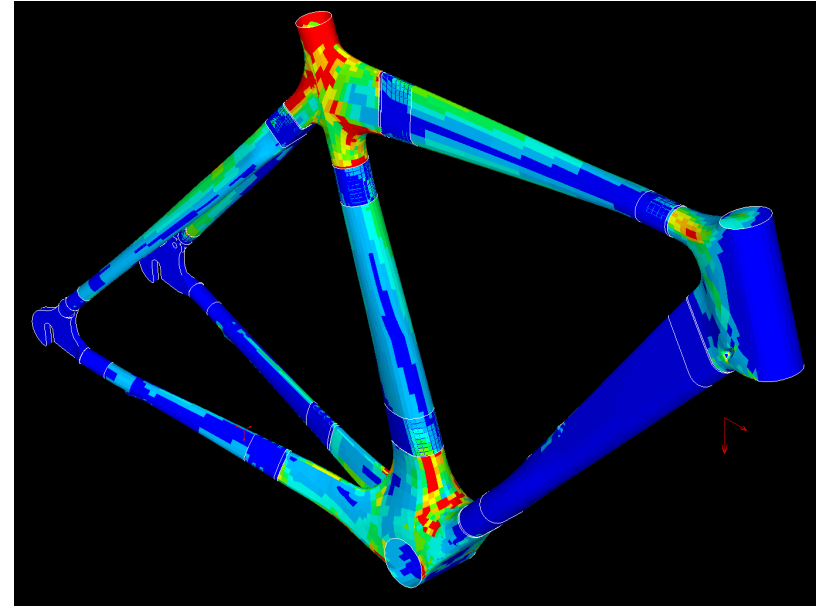
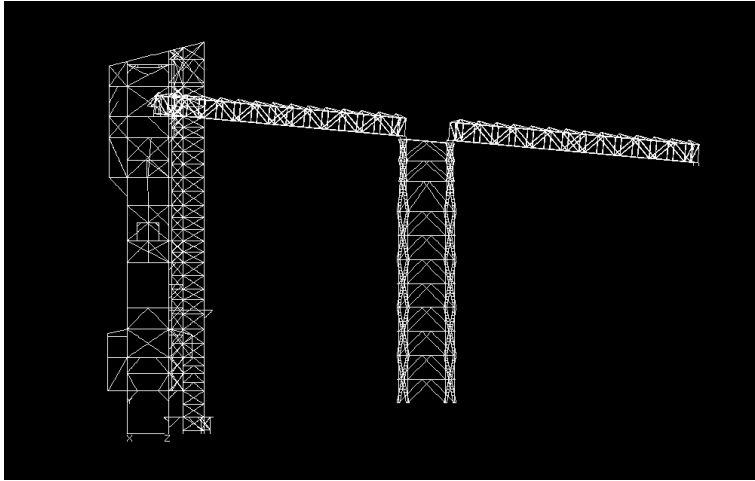
$$\tau_{xy} = -\frac{2}{\pi} \int_{-a}^a \frac{p(s)(x-s)y^2}{[(x-s)^2 + y^2]^2} ds - \frac{2}{\pi} \int_{-a}^a \frac{t(s)(x-s)^2 y}{[(x-s)^2 + y^2]^2} ds$$



# Method: Numerical Solution

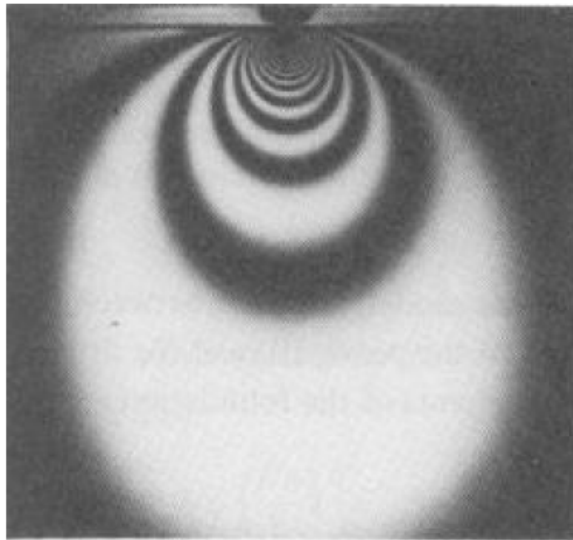
- **Finite element method (FEM):** can solve almost any problem in solid mechanics, provided you understand how to model your material
- **Finite difference method**
- **Boundary element method:** mostly useful for linear elastic problems
- **Atomistic methods:** integrate the equations of motion for atoms or molecules; use empirical constitutive equations; exceedingly small material volumes and short timescales

# Method: Numerical Solution

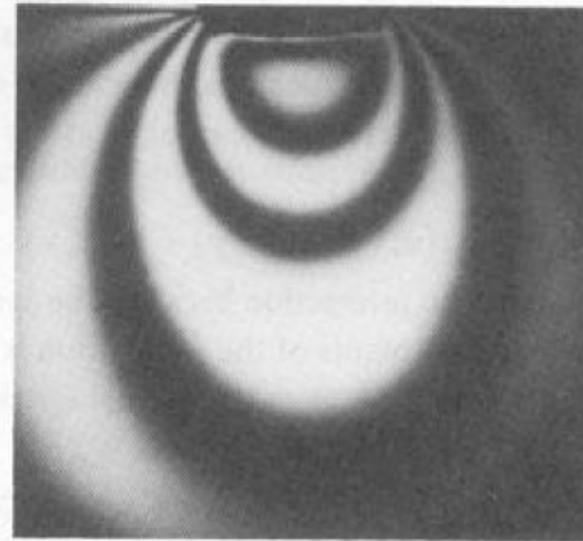


3.42 frame/sec

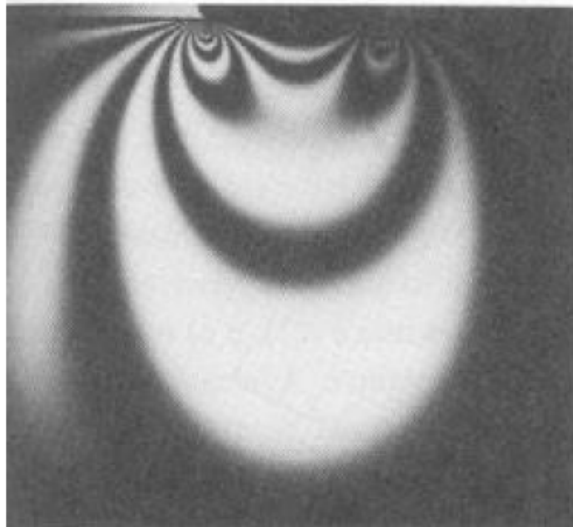
# Method: Experimental Solution



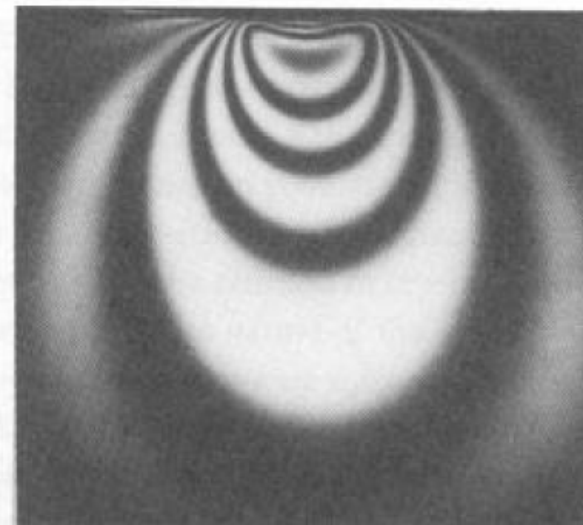
(Point Loading)



(Uniform Loading)



(Flat Punch Loading)



(Cylinder Contact Loading)

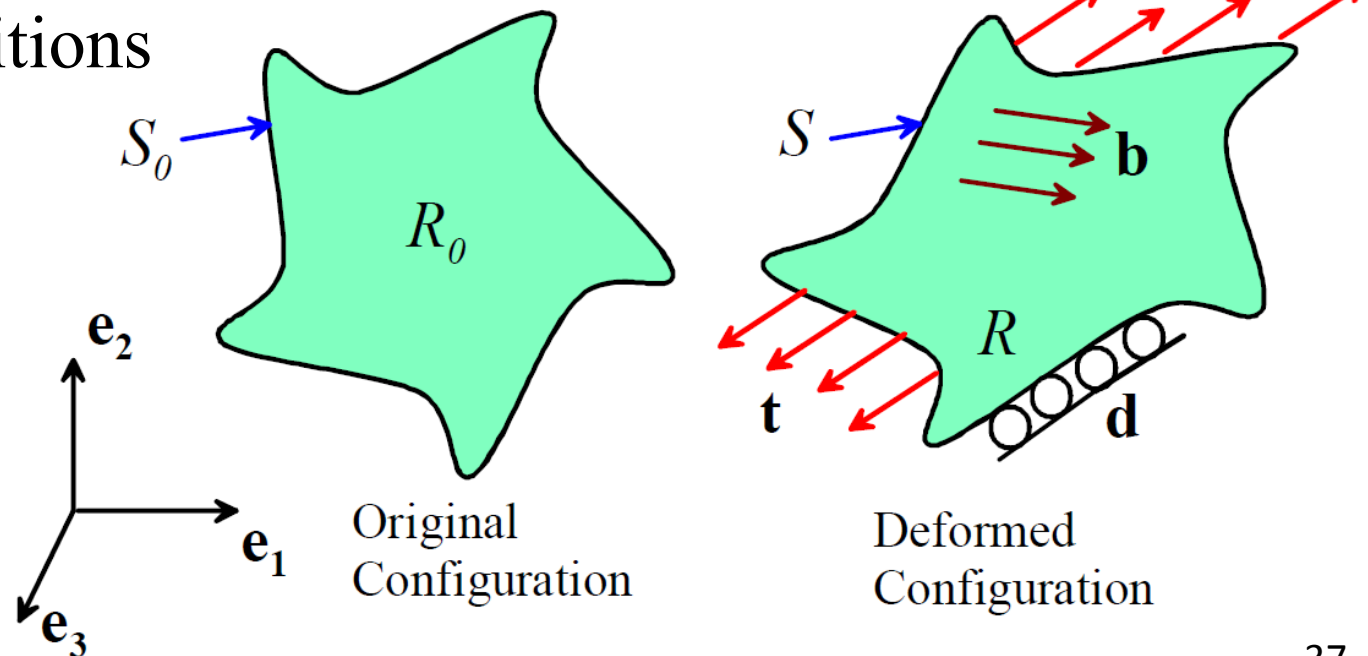
# Boundary Value Problems in Solid Mechanics

## Given:

- Geometry
- Material properties
- Applied loads
- Applied temperature/heat
- Initial conditions

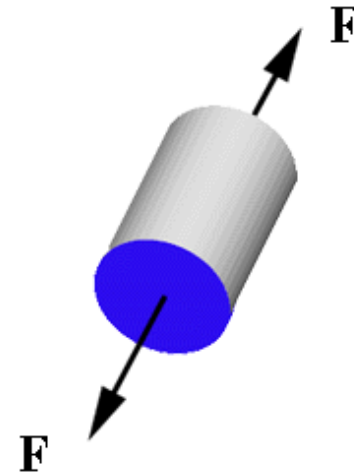
## Ask for:

- Deformed shape/stress
- Temperature
- Heat flux

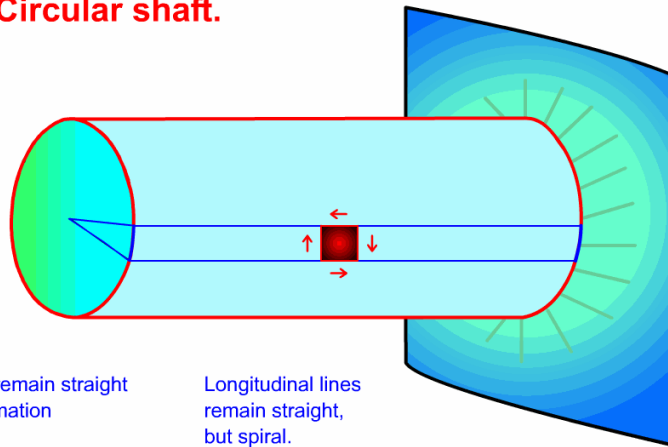


# What Did We Learn from Mechanics of Materials

- Tension of prismatic bars
- Torsion of circular shafts
- Bending of beams



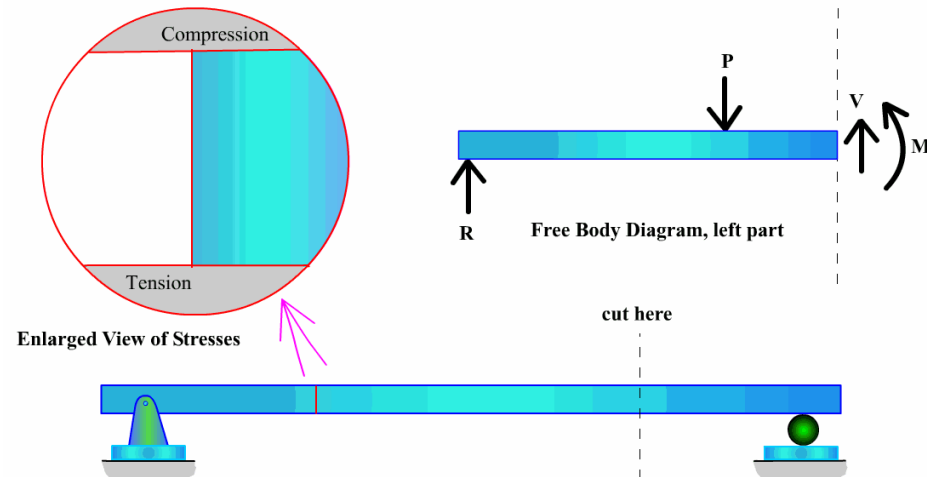
## Torsional Deformation of a Circular shaft.



Radial lines remain straight during deformation

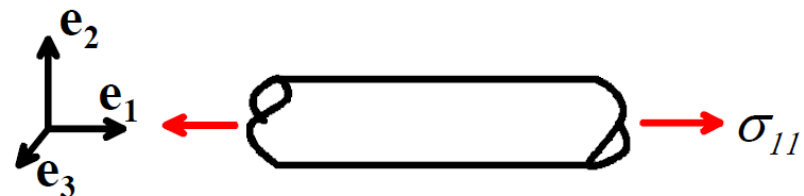
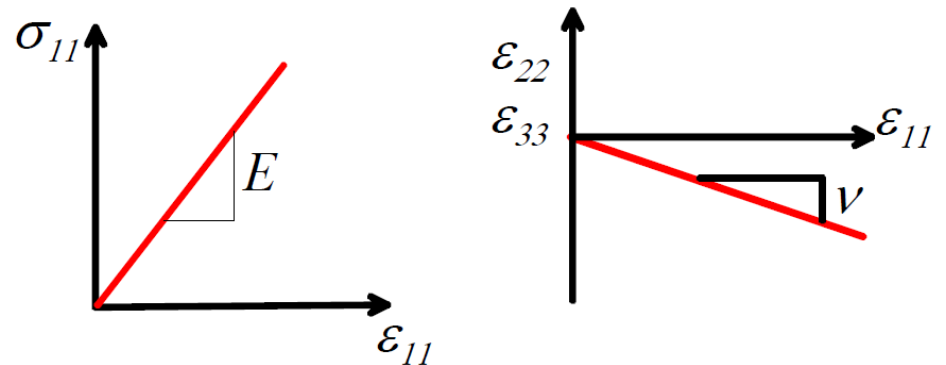
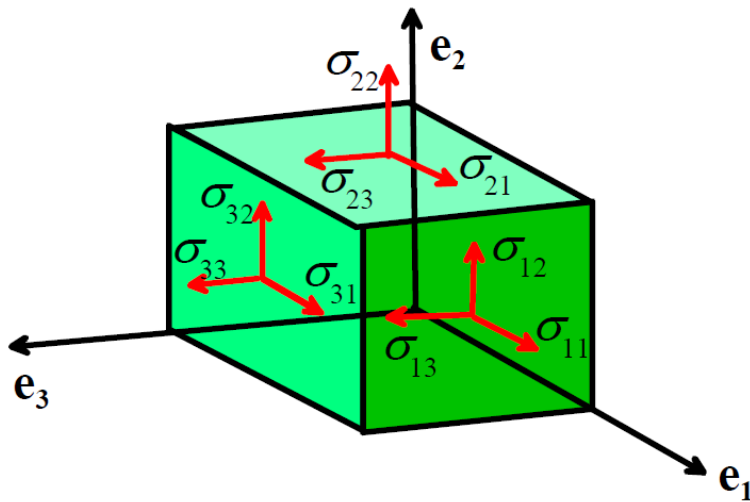
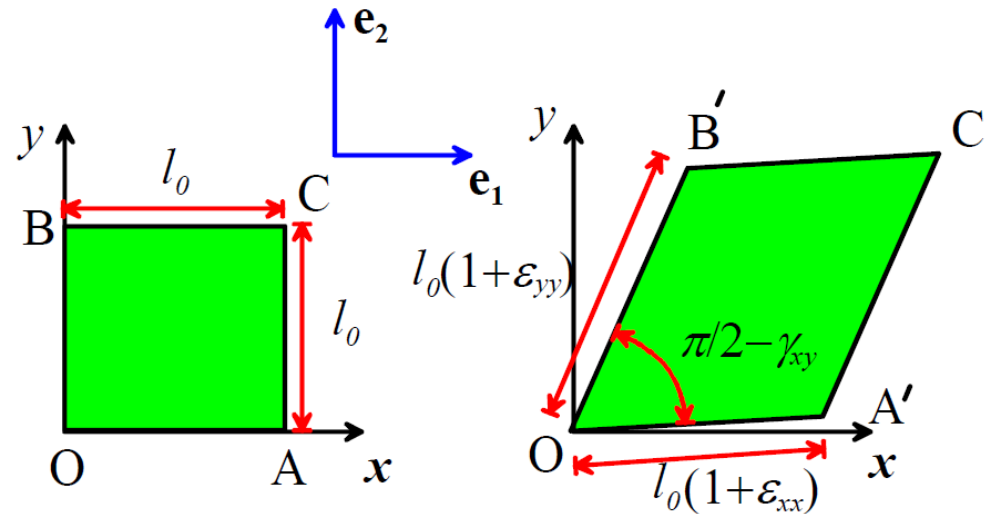
Longitudinal lines remain straight, but spiral.

Notice the deformation of the rectangular element when it is subjected to a torque



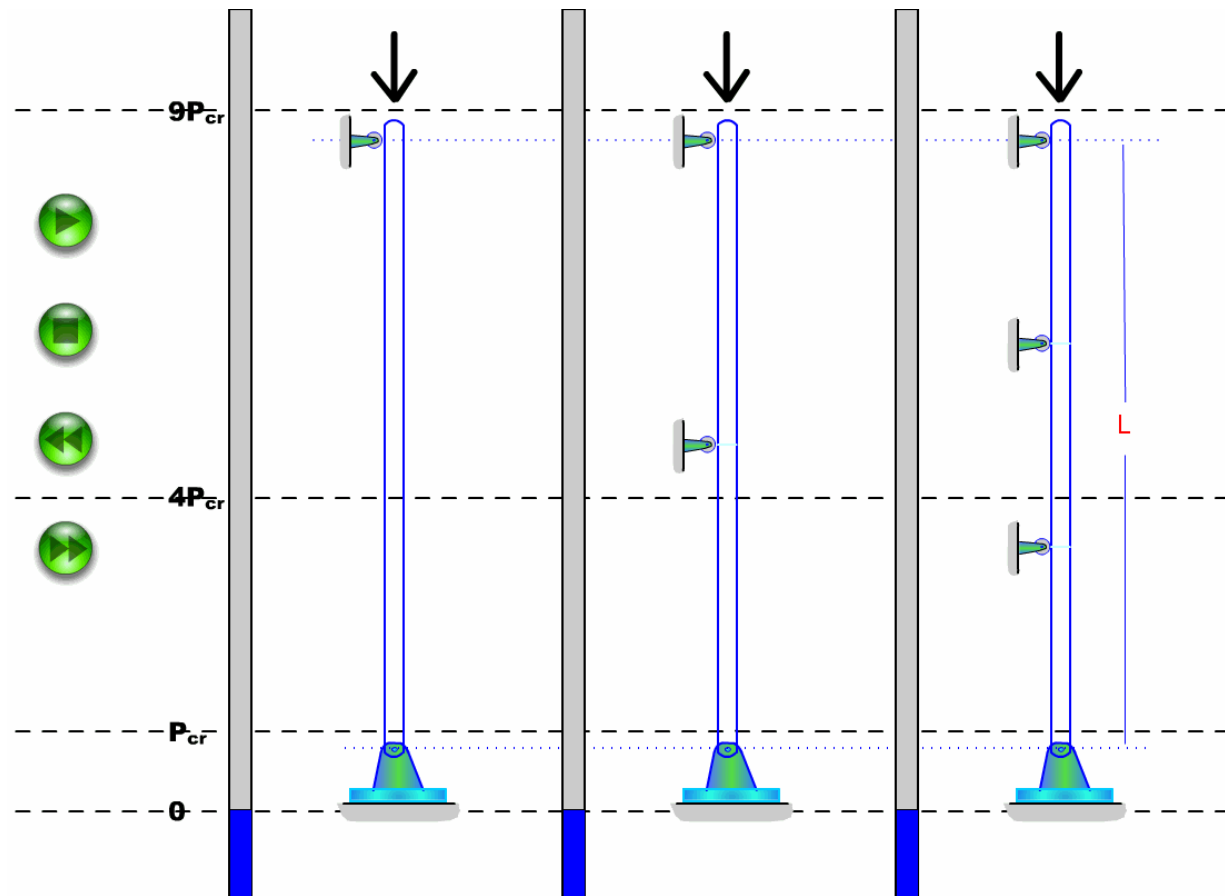
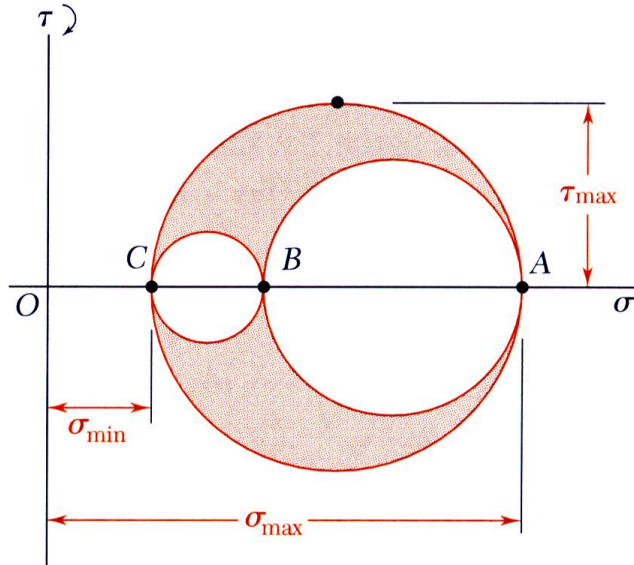
# What Did We Learn from Mechanics of Materials

- Deformation (kinematics)
- Stress (kinetics)
- Material behavior (Linear elastic)



# What Did We Learn from Mechanics of Materials

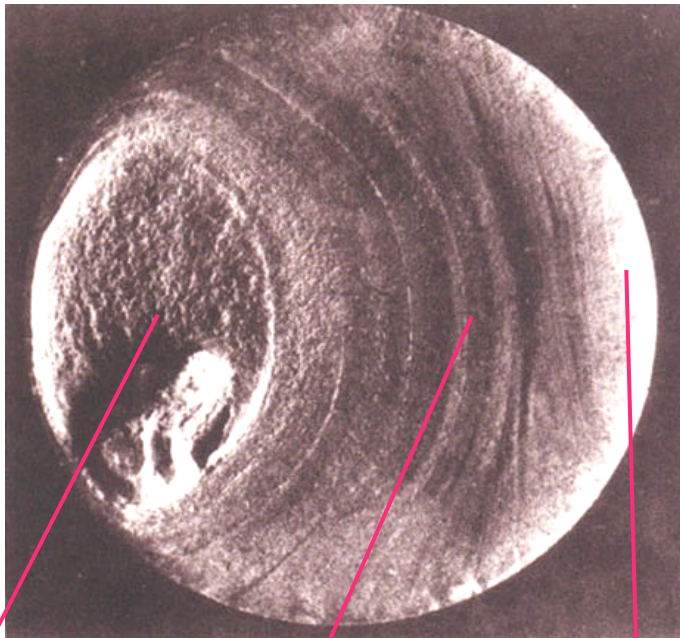
- Strength theory
- Stiffness condition
- Stability





# What Did We Learn from Mechanics of Materials

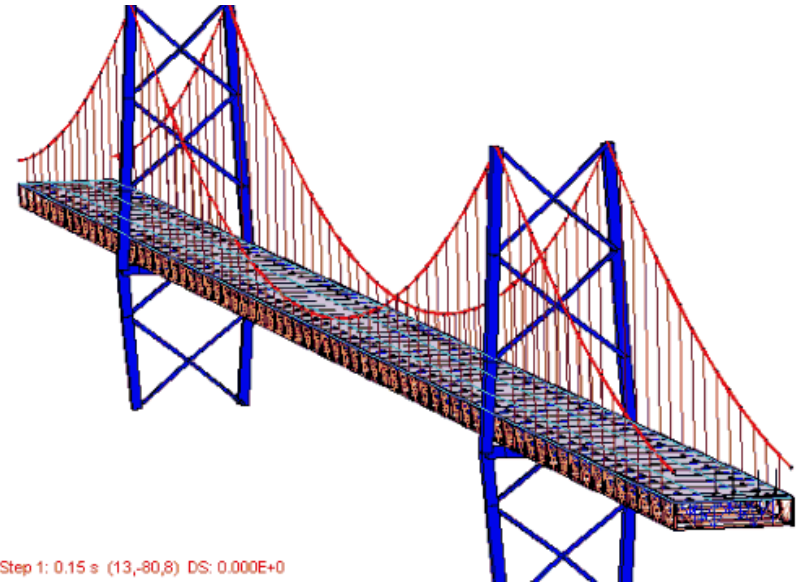
- Dynamic loading
- Cyclic loading



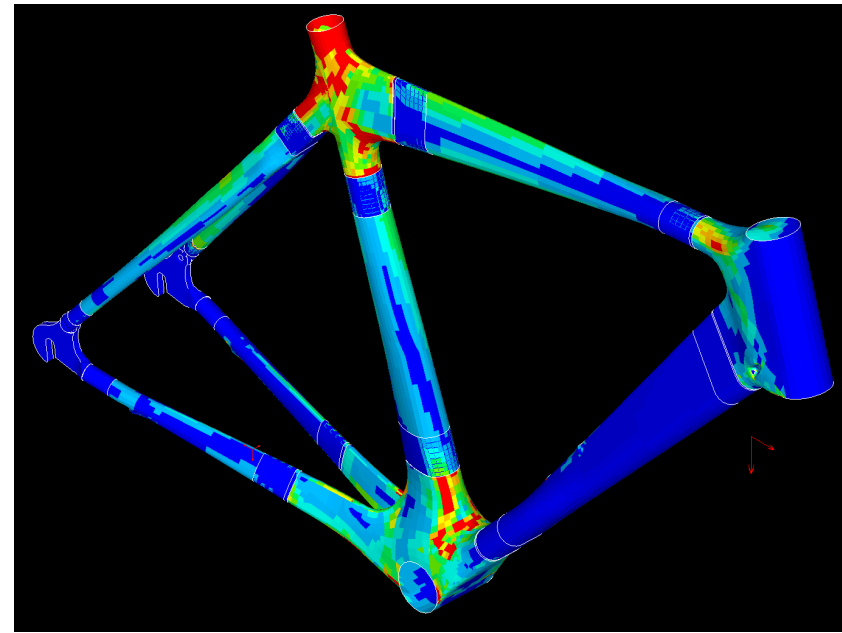
rough zone

fatigue origin

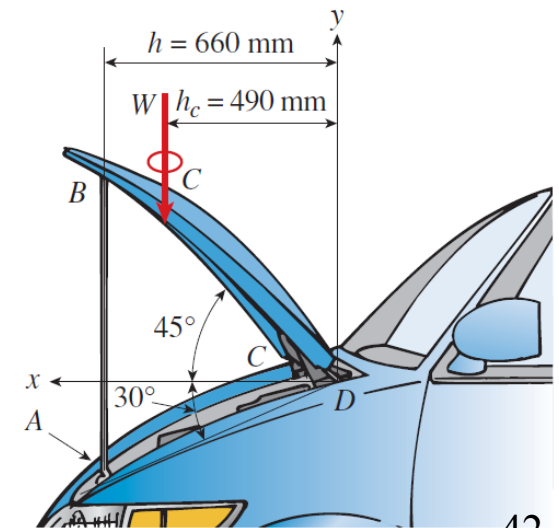
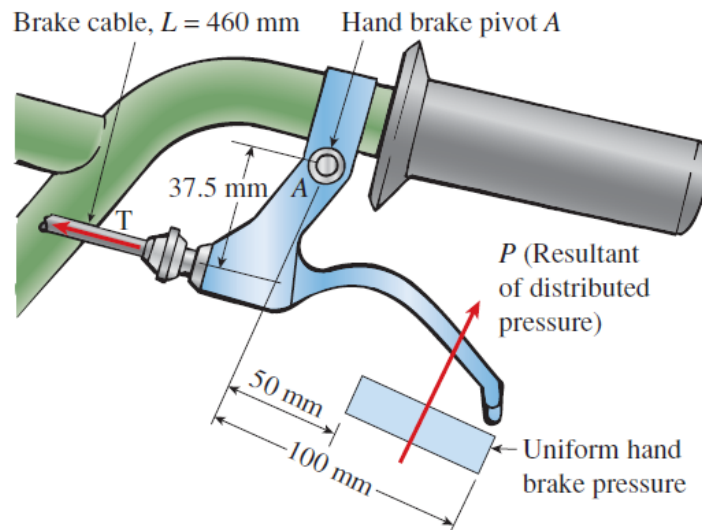
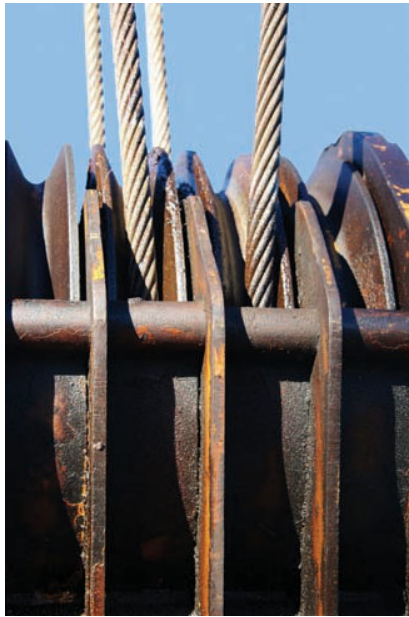
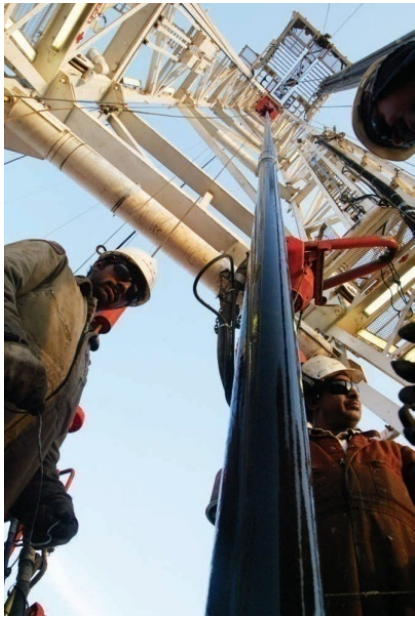
smooth zone



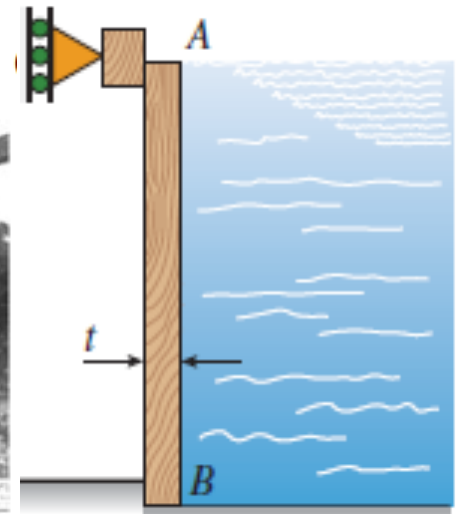
Step 1: 0.15 s (13,-80,8) DS: 0.000E+0



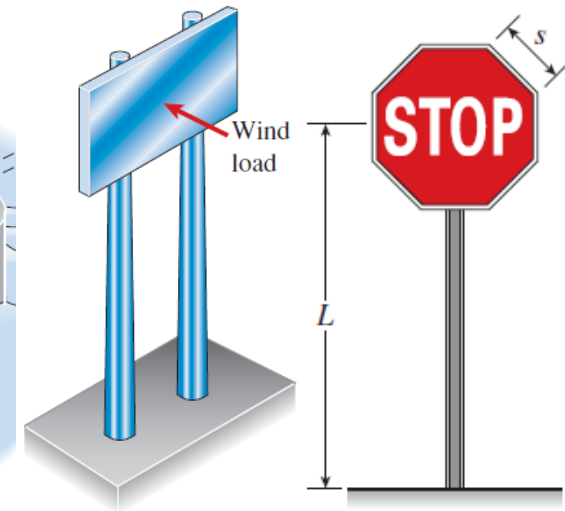
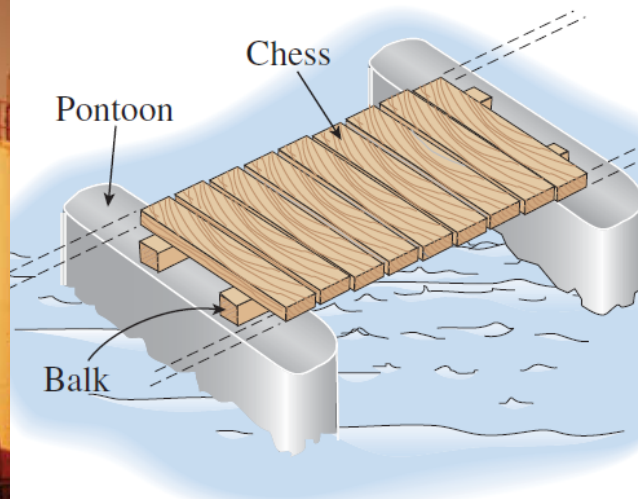
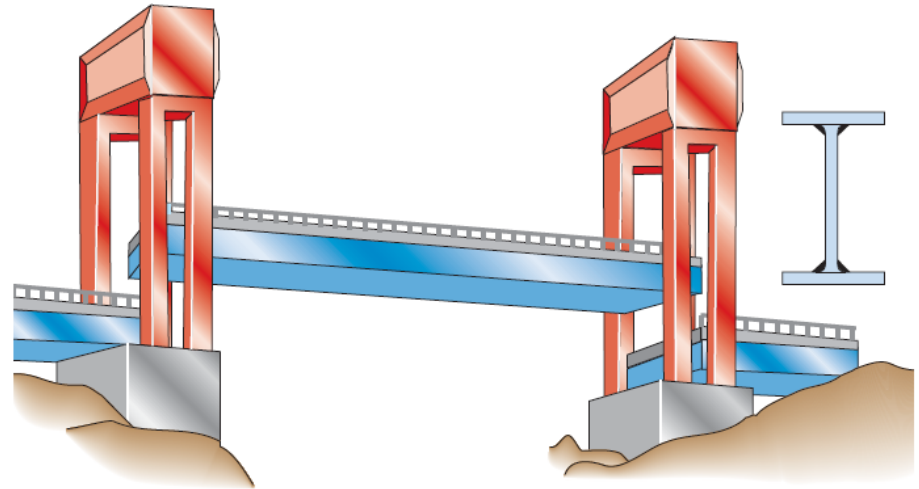
# What Did We Learn from Mechanics of Materials



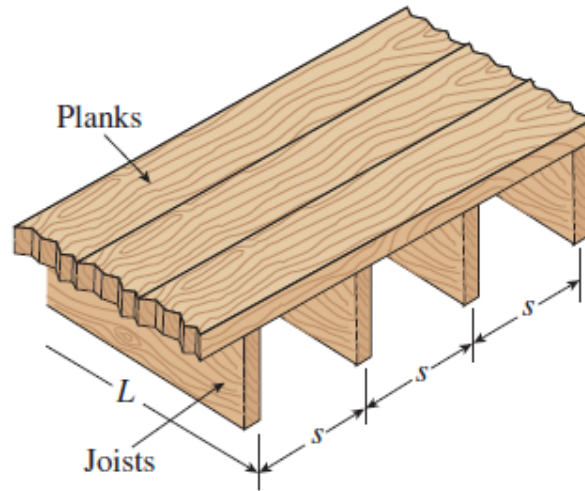
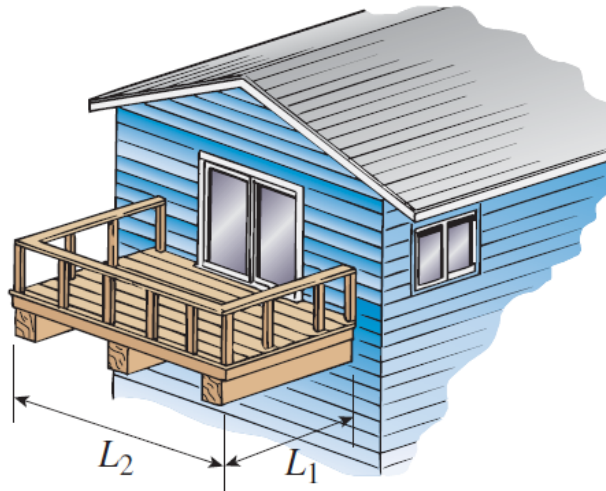
# What Did We Learn from Mechanics of Materials



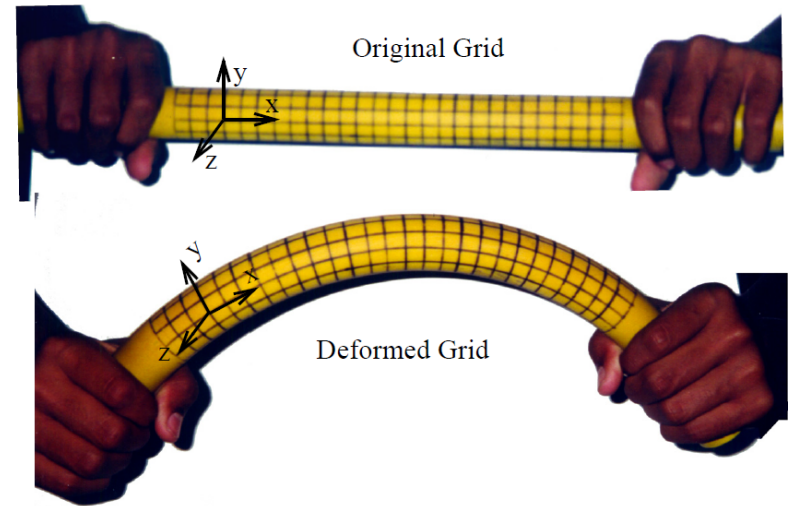
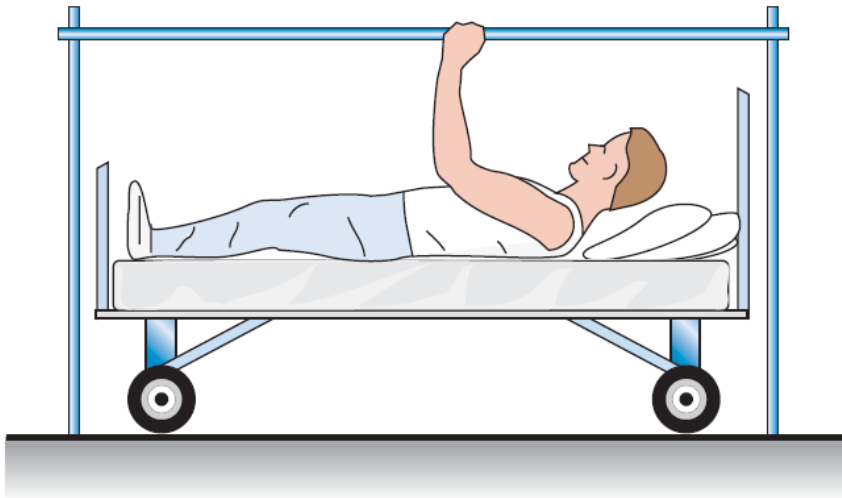
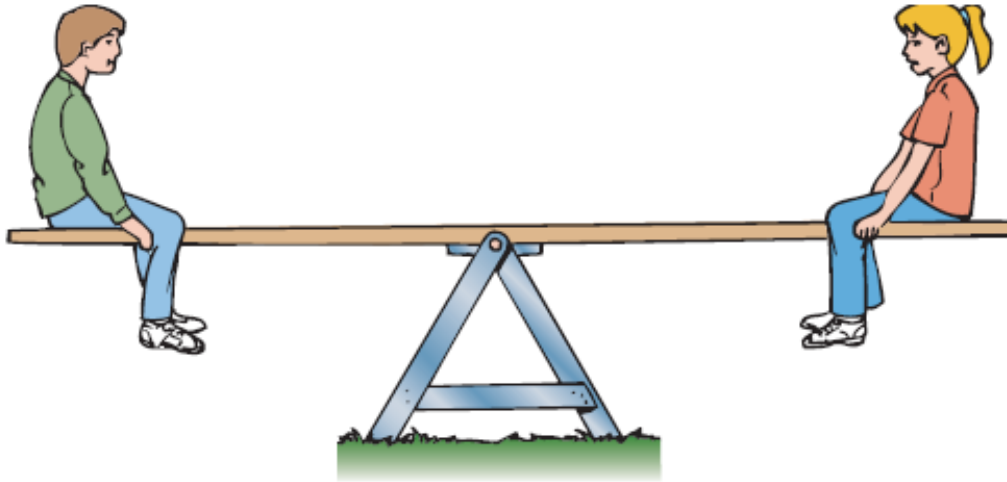
# What Did We Learn from Mechanics of Materials



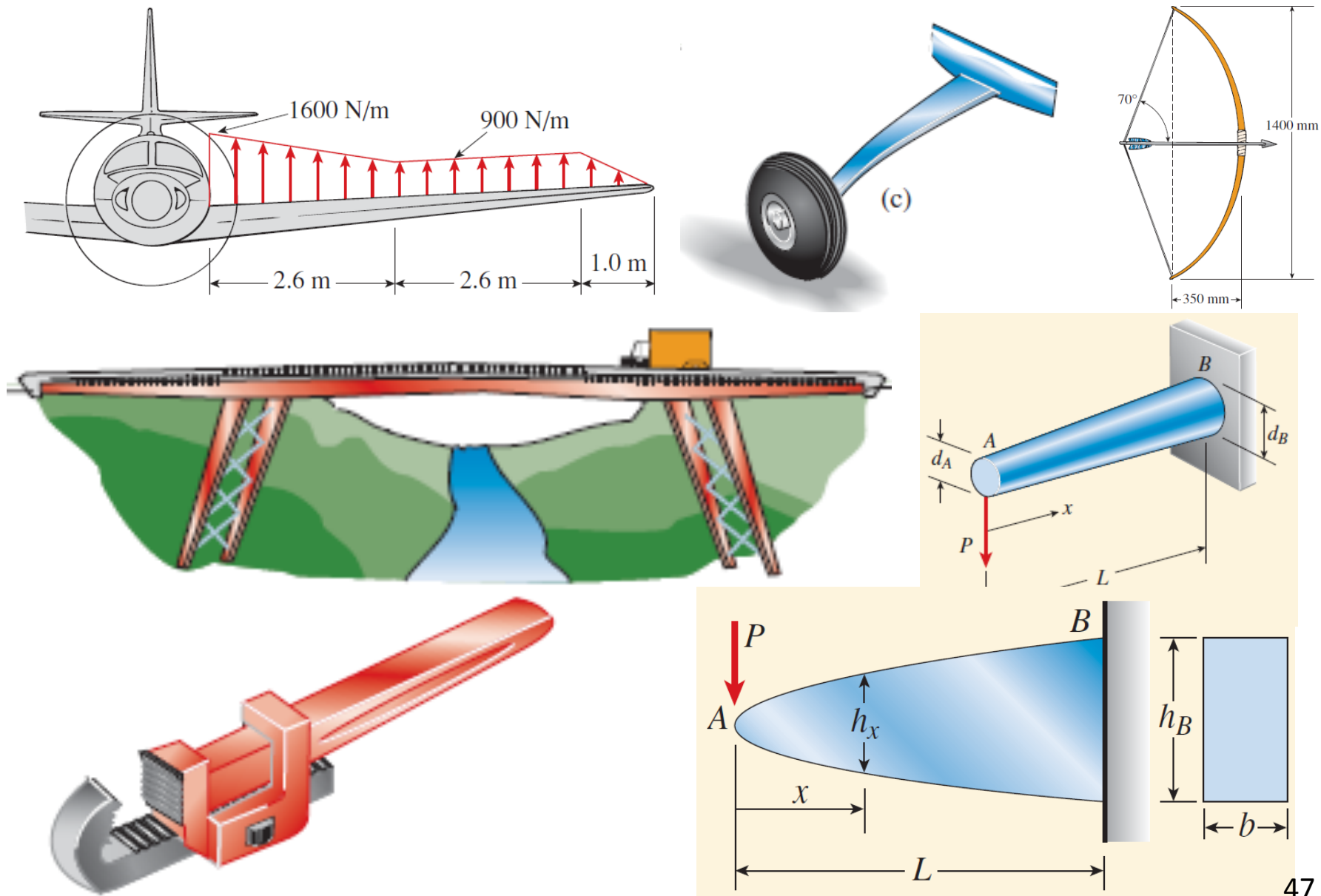
# What Did We Learn from Mechanics of Materials



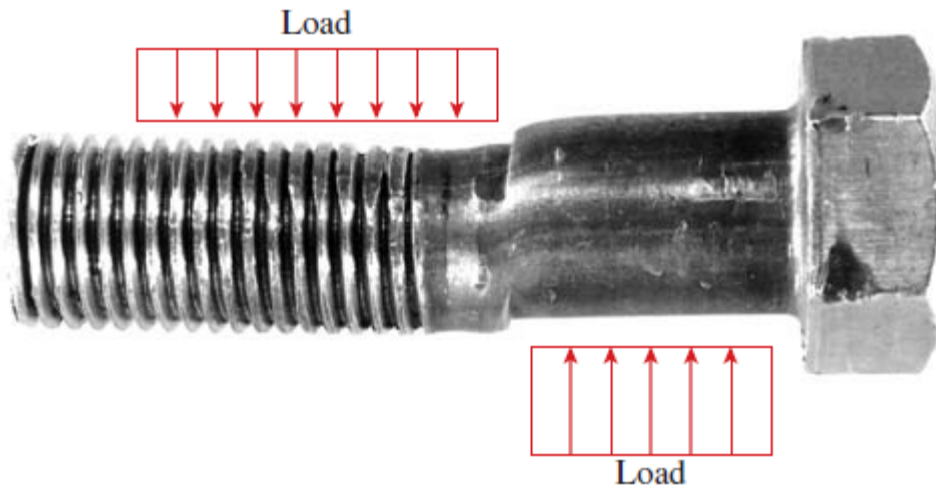
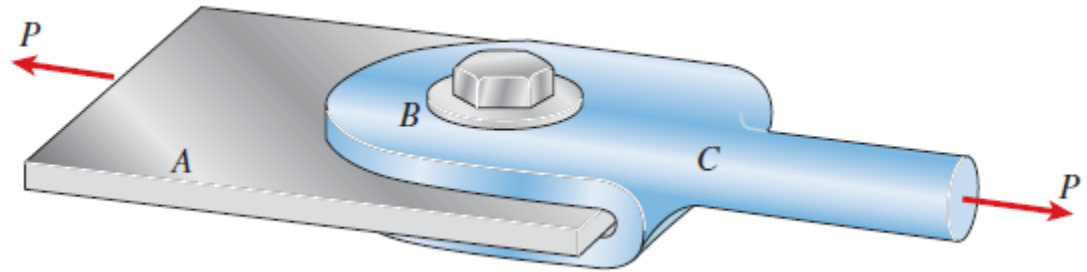
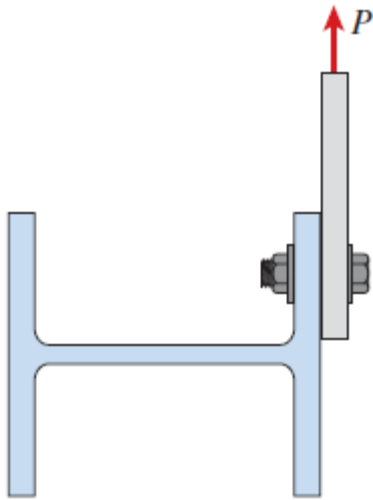
# What Did We Learn from Mechanics of Materials



# What Did We Learn from Mechanics of Materials



# What Did We Learn from Mechanics of Materials



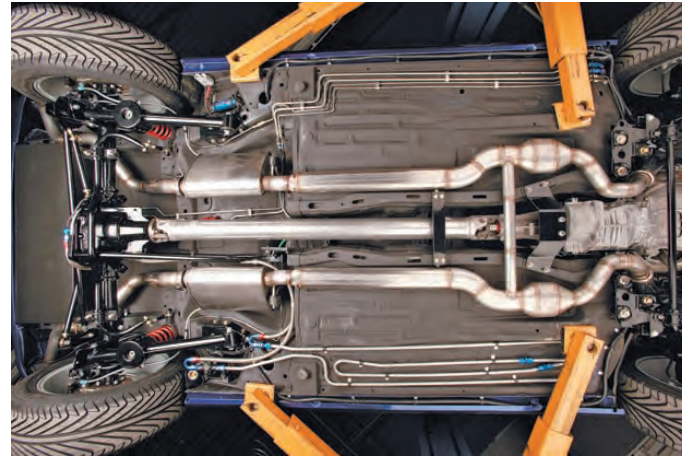
Failure of a bolt in single shear



# What Did We Learn from Mechanics of Materials



Power generation shaft



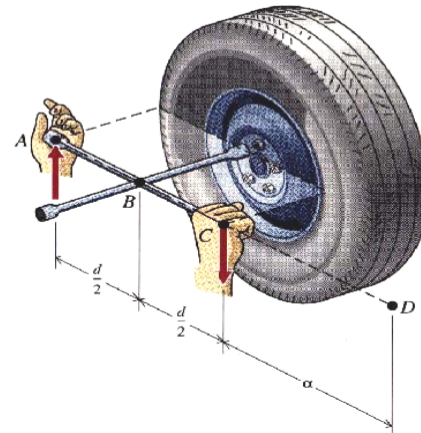
Automotive power train shaft



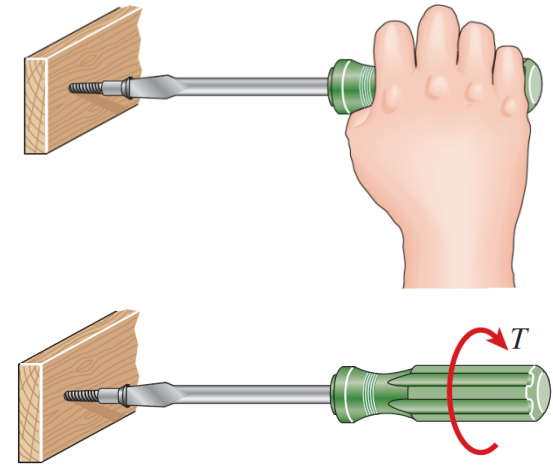
Ship drive shaft



Complex crank shaft

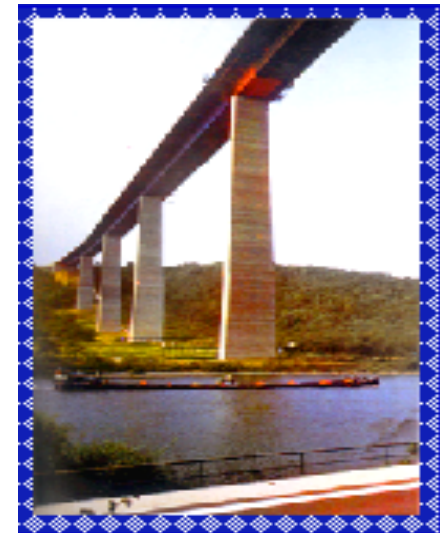


Tire shift drive

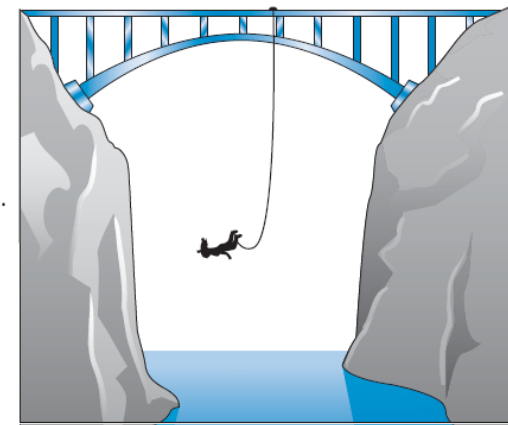
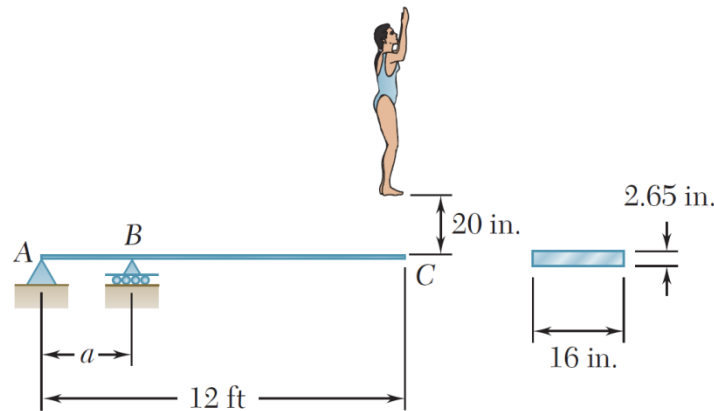
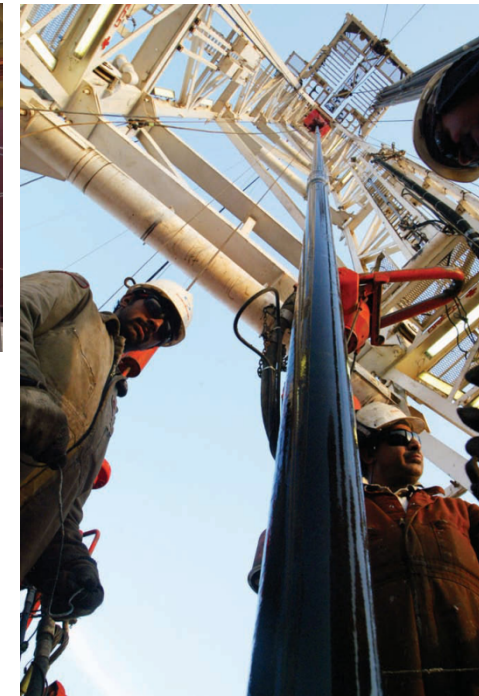
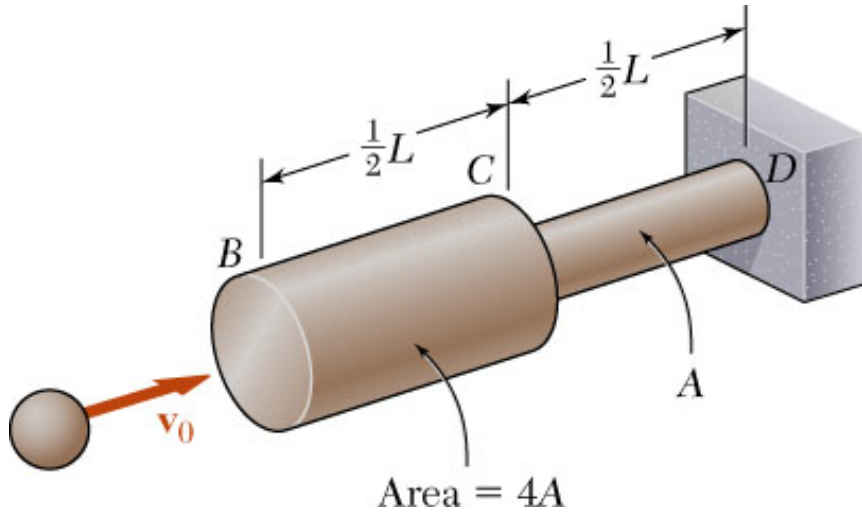


Screwdriver

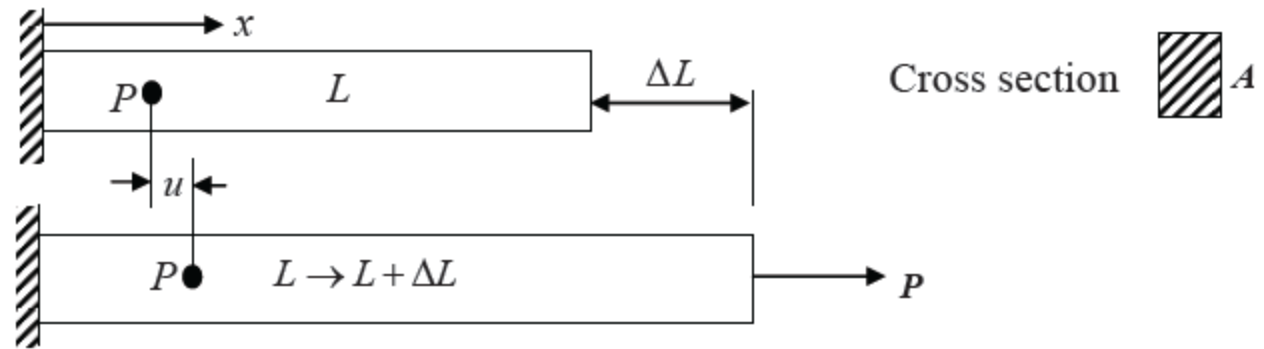
# What Did We Learn from Mechanics of Materials



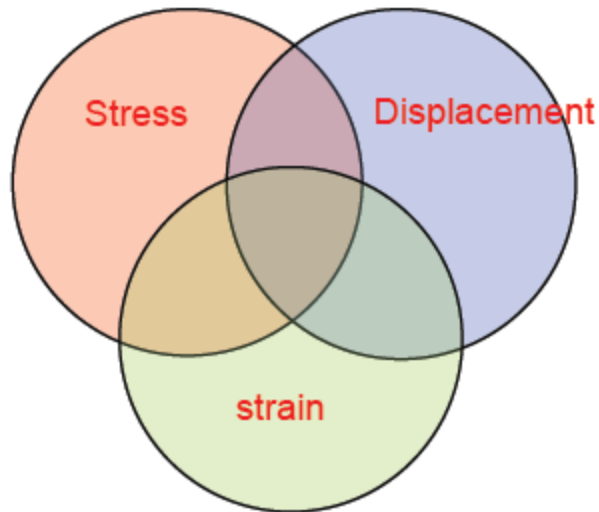
# What Did We Learn from Mechanics of Materials



# What Did We Learn from Mechanics of Materials



Solid mechanics



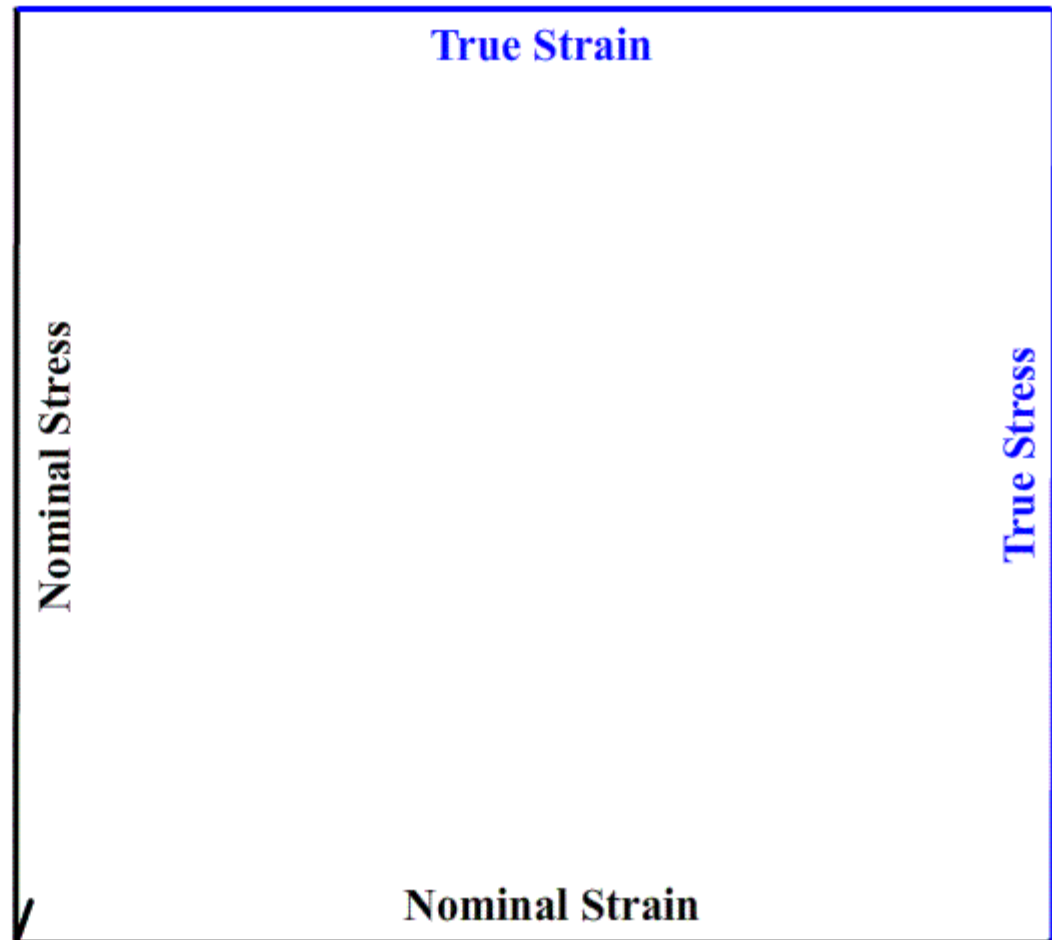
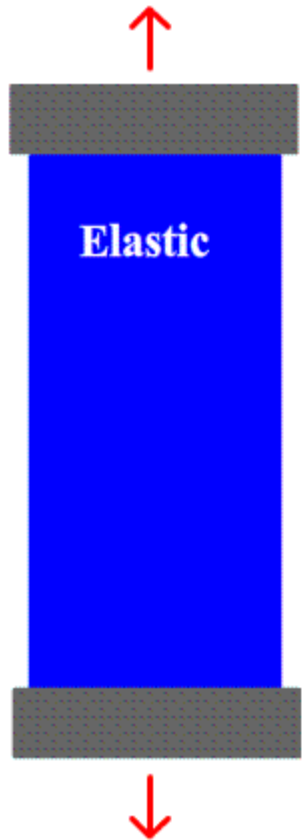
Displacement:  $u = u(x)$  = the distance over which a material point has moved during deformation

Stress:  $\sigma = P / A$  = force per unit area

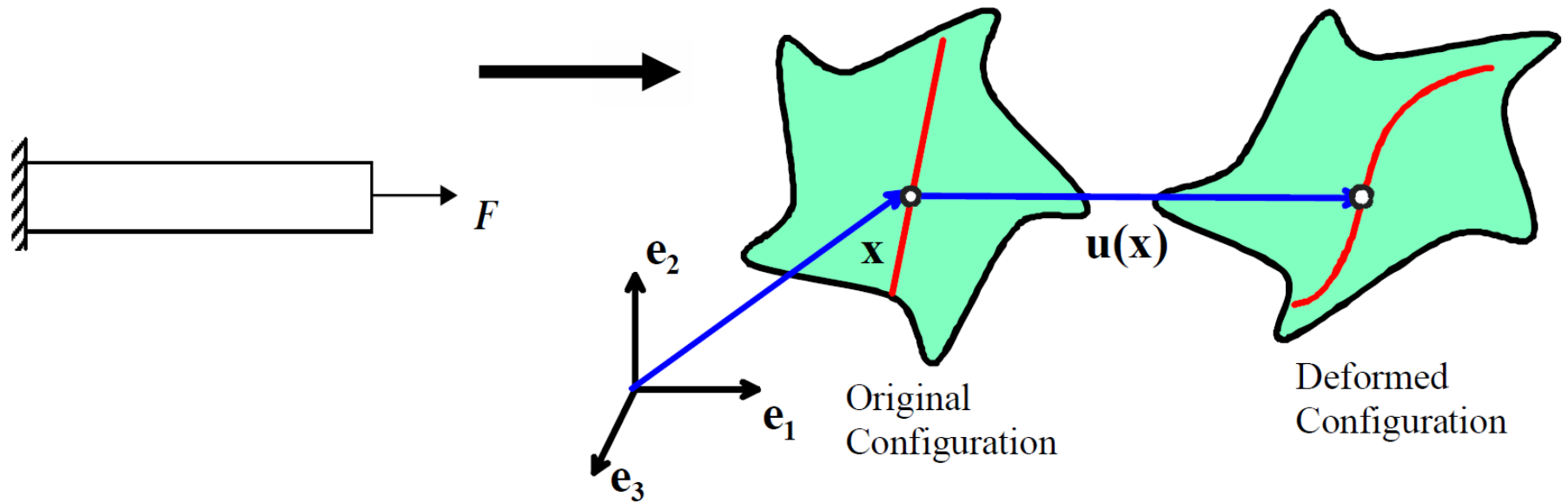
Strain:  $\varepsilon = \Delta L / L$  = percent of elongation

# Generalization of Concepts in Mechanics of Materials

## Strain Hardening Exponent for a Typical Ductile Material (Not to Scale)



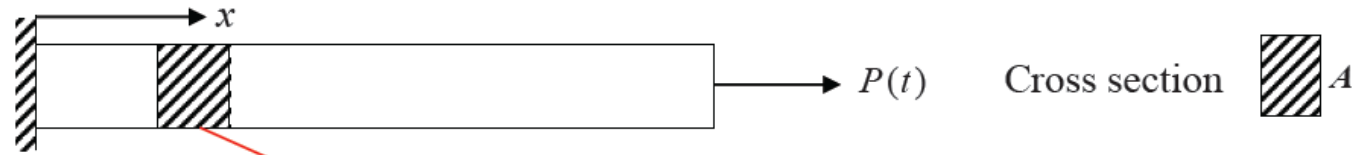
# Generalization of Concepts in Mechanics of Materials



- Displacement: vectors
- Stress and strain: tensors
- Kinematics + Kinetics
- Material behaviors
- Boundary conditions
- A unified mathematical framework is needed to understand these concepts in a fully three dimensional continuum solid body. ♪

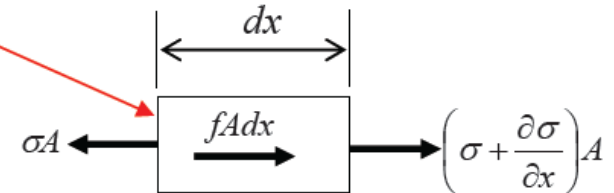
# A 1D Example: Momentum Balance

Stress  $\sigma = \sigma(x, t)$

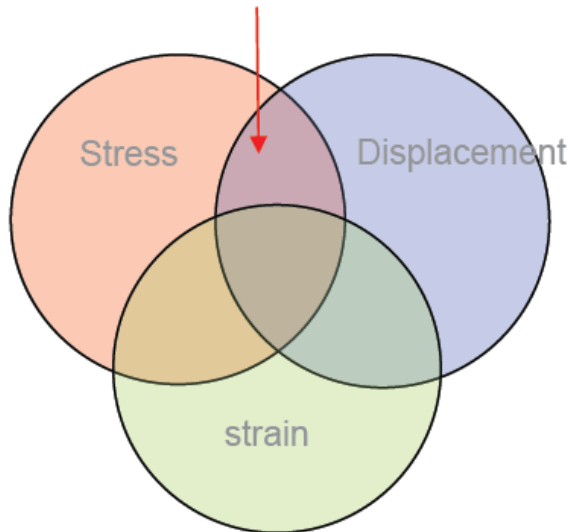


Solid mechanics

Forces on a small segment:



Momentum balance



Newton's law:  $\sum F = ma$

$$\sum F = \left( \sigma + \frac{\partial \sigma}{\partial x} dx \right) A - \sigma A + f A dx = \left( \frac{\partial \sigma}{\partial x} + f \right) A dx$$

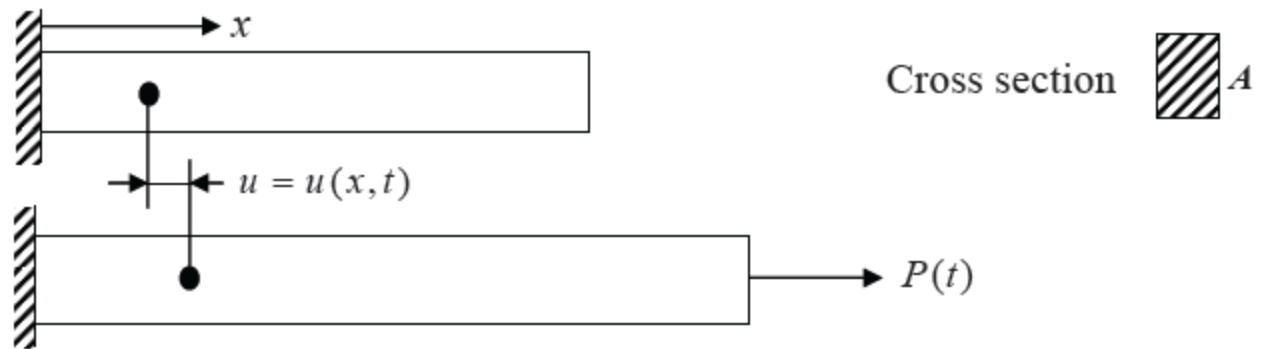
$$m = \rho A dx$$

Equilibrium equation:

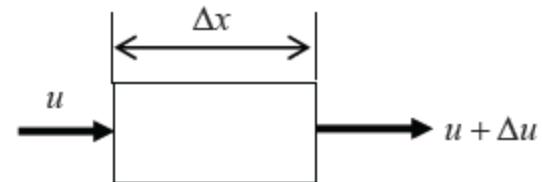
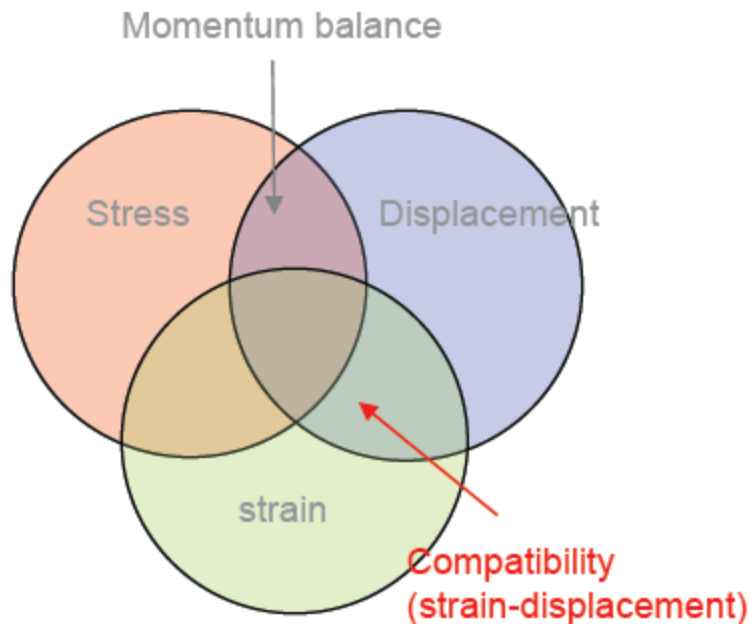
$$a = \frac{\partial^2 u}{\partial t^2}$$

$$\sum F = ma \rightarrow \boxed{\frac{\partial \sigma}{\partial x} + f = \rho \frac{\partial^2 u}{\partial t^2}}$$

# A 1D Example: Kinematics



## Solid mechanics



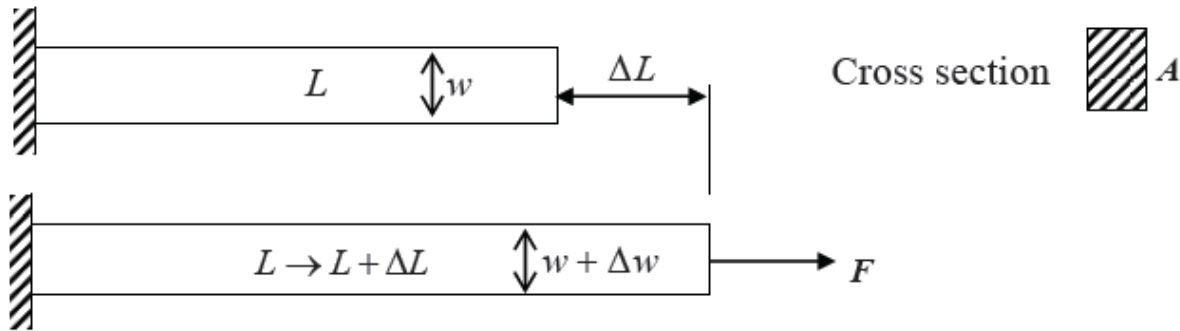
$$\varepsilon = \lim_{\Delta x \rightarrow 0} \frac{\Delta u}{\Delta x} = \frac{\partial u}{\partial x}$$

Strain is related to displacement gradient

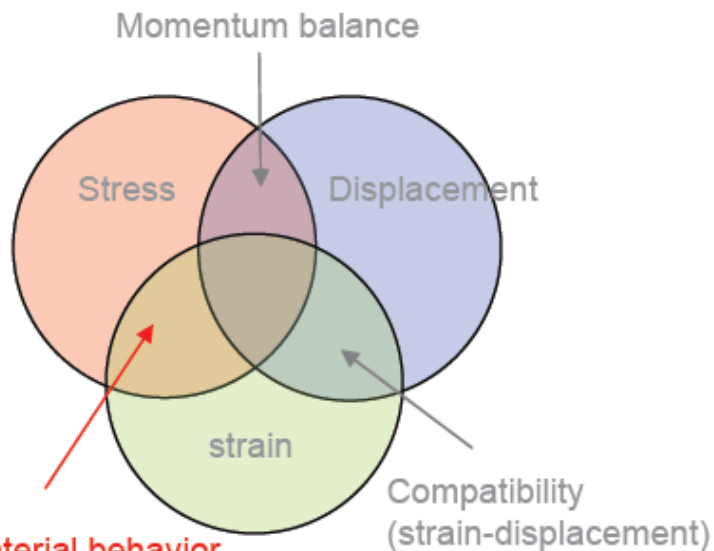
$u = \text{constant}$   $\longrightarrow$  rigid body motion



# A 1D Example: Material Behavior



## Solid mechanics



$$\sigma = F/A \quad \varepsilon = \Delta L/L \quad \varepsilon' = \Delta w/w < 0$$

**Empirical/experimental observations for most solid materials under small deformation:**

Hooke's law:  $\sigma = E\varepsilon$  or  $\varepsilon = \frac{\sigma}{E}$

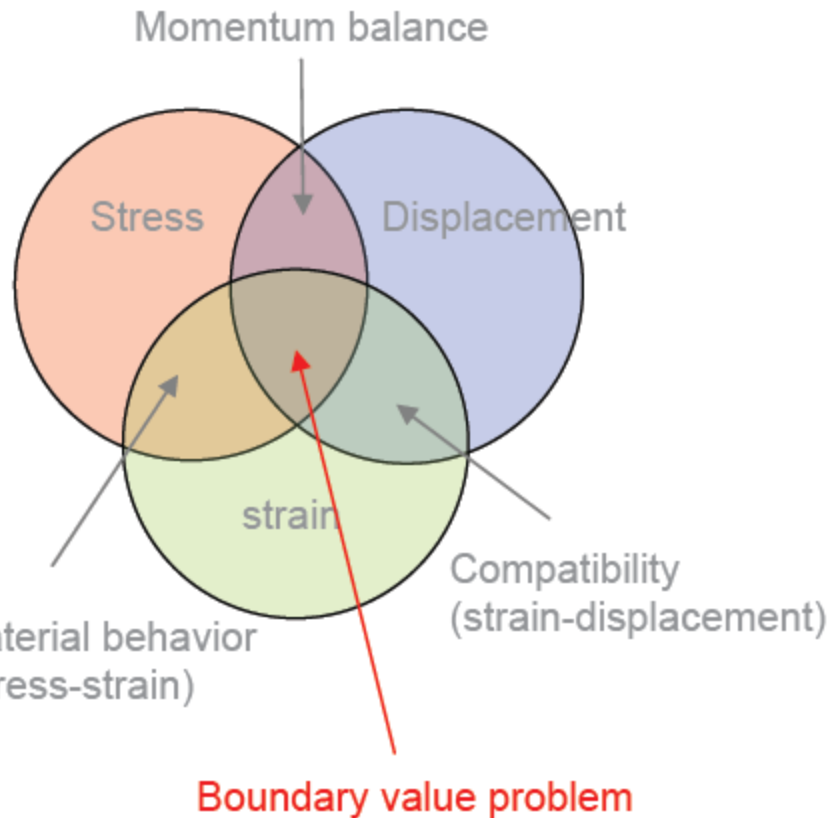
$E$  = Young's modulus

Poisson ratio:  $\varepsilon' = -\nu\varepsilon$

**Remark:** When deformation increases, the material may undergo plastic deformation, creep, or even failure.

# A 1D Example: BVP Formulation

## Solid mechanics



Equilibrium equation:  $\frac{\partial \sigma}{\partial x} + f = \rho \frac{\partial^2 u}{\partial t^2}$

Strain-displacement relation:  $\varepsilon = \frac{\partial u}{\partial x}$

Hooke's law:  $\sigma = E\varepsilon$

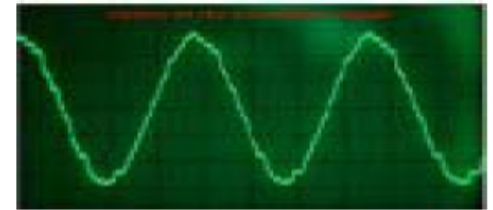
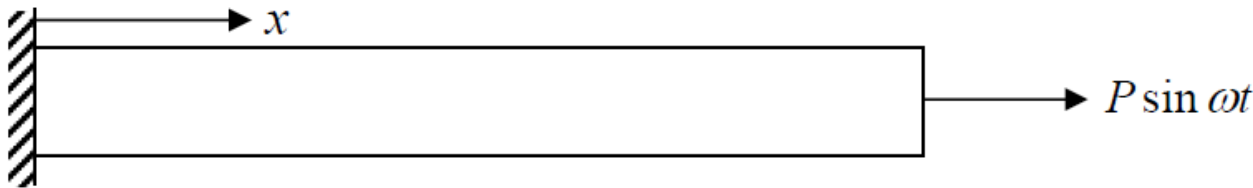
Unknowns:  $u, \varepsilon, \sigma$

Boundary conditions:

Displacement BC:  $u = \hat{u} @ \text{boundaries}$

Traction BC:  $\sigma = \hat{\sigma} @ \text{boundaries}$

# A 1D Example: Sinusoidal Traction BCs



$$\frac{\partial \sigma}{\partial x} + f = \rho \frac{\partial^2 u}{\partial t^2} \quad \varepsilon = \frac{\partial u}{\partial x} \quad \sigma = E \varepsilon = E \frac{\partial u}{\partial x}$$

$$\longrightarrow \boxed{E \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}}$$

$$\begin{aligned} \text{BCs: } \quad u &= 0 & @ \quad x &= 0 \\ E \frac{\partial u}{\partial x} &= P \sin \omega t & @ \quad x &= L \end{aligned}$$

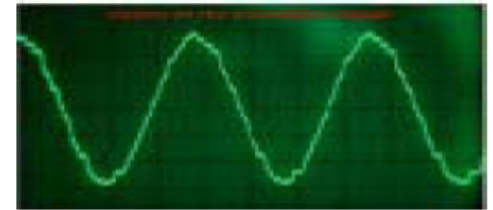
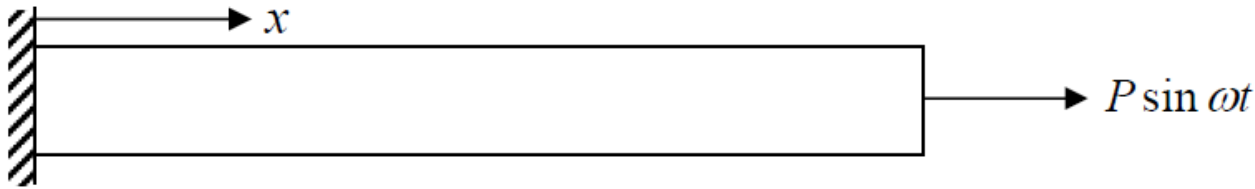
Method of solution:  $u = U(x) \sin \omega t$  (separation of variable method in PDE)

$$\boxed{EU''(x) = -\rho \omega^2 U(x)} \quad \text{BCs: } U(0) = 0, \quad EU'(L) = P$$

$$\text{Solution: } U(x) = P \left( \sin \sqrt{\frac{\rho}{E}} \omega x \right) / \left( \sqrt{\rho E} \omega \cos \sqrt{\frac{\rho}{E}} \omega L \right)$$

$$u(x,t) = P \left( \sin \sqrt{\frac{\rho}{E}} \omega x \sin \omega t \right) / \left( \sqrt{\rho E} \omega \cos \sqrt{\frac{\rho}{E}} \omega L \right), \quad \sigma(x,t) = P \sin \omega t \left( \cos \sqrt{\frac{\rho}{E}} \omega x / \cos \sqrt{\frac{\rho}{E}} \omega L \right)$$

# A 1D Example: Resonance



$$\sigma(x,t) = \frac{P \sin \omega t \cos \sqrt{\frac{\rho}{E}} \omega x}{\cos \sqrt{\frac{\rho}{E}} \omega L}$$

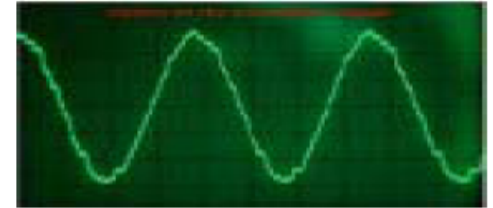
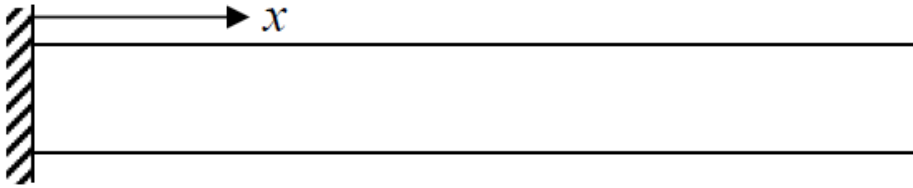
Large stresses develop when

$$\cos \sqrt{\frac{\rho}{E}} \omega L = 0 \quad \sqrt{\frac{\rho}{E}} \omega L = \left(n + \frac{1}{2}\right) \pi \quad n = \text{integer}$$

$$\omega = \left(n + \frac{1}{2}\right) \frac{\pi}{L} \sqrt{\frac{E}{\rho}} \longrightarrow \text{Structural failure}$$

- Shattering of glass at certain high pitch
- Ultrasonic destruction of kidney stone

# A 1D Example: Natural Vibration



$$E \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2}$$

**BCs:**  $u = 0$  @  $x = 0$   
 $\sigma = E \frac{\partial u}{\partial x} = 0$  @  $x = L$

**Assume:**  $u = U(x) \sin \omega t$

$$EU''(x) = -\rho\omega^2 U(x)$$

**BCs:**  $U(0) = 0, \quad U'(L) = 0$

**There are solutions of type:**  $U(x) = A \sin \sqrt{\frac{\rho}{E}} \omega x$  when  $\omega_N = \left(n + \frac{1}{2}\right) \frac{\pi}{L} \sqrt{\frac{E}{\rho}}$

$$u(x, t) = A \sin \frac{(2n+1)\pi x}{2L} \sin \omega t$$

- Resonance of a structure occurs when the frequency of applied force matches the natural vibrating frequency of the structure, i.e.  $\omega = \omega_N$ .

# Course Outline

- Introduction (导论)
- Mathematical background (数学基础)
- Strains in a solid/Kinematics (应变度量/运动学)
- Stress in a solid/Kinetics (应力度量/动力学)
- Mechanical behavior of solids (固体本构)
- Simple boundary value problems (BVPs) (简单变值问题)
- BVPs for elastic solids (弹性力学边值问题)
- BVPs for plastic solids (塑性力学边值问题)
- Failure modes in solids (固体失效模式)
- Variational methods (变分法)
- The finite element method (有限元法)

# References

- **Textbook:**

- Allan Bower, Applied Mechanics of Solids, CRC Press, 2010. (available on <http://solidmechanics.org/> for free)

- **Highly recommended references:**

- T. Belytschko, W.K. Liu, B. Moran and K.I. Elkhodary, Nonlinear Finite Elements for Continua and Structures, 2<sup>nd</sup> Ed., John Wiley & Sons, 2014.
- Y.C. Fung, A First Course in Continuum Mechanics, 3<sup>rd</sup> Ed., Prentice-Hall, 1994.♪
- Y.C. Fung and P. Tong, Classical and Computational Solid Mechanics, World Scientific, 2001.
- L. Malvern, An Introduction to the Mechanics of a Continuous Medium, Prentice-Hall, 1969.♪

# Grading and Collaboration Policies

- Assignment category      Percentage
- Homework                      30%
- Project                         30%
- Final                            40%
- **Collaboration policy:** We encourage discussions on homework and computer assignments: you can learn a lot from working with a group. This means that you are permitted to discuss homework problems and computer assignments with classmates, and are permitted to seek help from other students if you run into difficulties. However, material submitted for grading should represent independent work of its author. Any work done in collaboration should be clearly marked as such. It is not acceptable to copy the work of other students, and it is not acceptable for two students to submit identical copies of any part of an assignment.



# Computing Project

---

1. Choose a problem to solve.
2. Write a short proposal that defines the problem to be solved, describes what will be calculated, outlines briefly how the calculation will be done.
3. Submit the proposal via email and obtain approval from the instructor.
4. Set up a theoretical or finite element model and perform the calculations, analyses, and discussions.
5. Write a formal report no more than 10 pages that summarizes the results.
6. Give a short oral presentation to the rest of the class.

# Greek Letters

Letter	Greek	English	Sounds As	Phonetic	Trsl.
A α	ἄλφα	āphā	ahl-fah	a in father (long) a in dad (short)	a
B β	βῆτα	bētā	bay-tah	b in ball	b
Γ γ	γάμμα	gāmmā	gahm-mah	g in got	g
Δ δ	δέλτα	dēltā	dell-tah	d in dog	d
E ε	ἒ ψιλόν	ēpsilōn	eh-pseeh-lawn	e in net	e
Z ζ	ζῆτα	zētā	zay-tah	z in gaze (initial) dz in adz (medial)	z
H η	ἦτα	ētā	ay-tah	e in obey	ē
Θ θ	θῆτα	thētā	thay-tah	th in this	th
I ι	ἰῶτα	iōtā	yi-oh-tah	i in machine (long) i in hit (short)	i
K κ	κάππα	kāppā	kap-pah	k in kin	k
Λ λ	λάμβδα	lāmbdā	lahm-dah	l in lot	l
M μ	μῦ	mū	mew	m in man	m
N ν	νῦ	nū	new	n in new	n

# Greek Letters

Letter	Greek	English	Sounds As	Phonetic	Trsl.
Ξ ξ	ξῖ	xsī	x-see	<b>x</b> in <b>axe</b>	<b>x</b>
Ο ο	ὀ μικρόν	ōmīkron	au-me-krahn	<b>ough</b> in <b>ought</b>	<b>o</b>
Π π	πί	pī	peeh	<b>p</b> in <b>party</b>	<b>p</b>
Ρ ρ	ῥῶ	rhō	hrow	<b>r</b> in <b>ride</b> <b>rh</b> in <b>rhino</b> (aspirate)	<b>r</b>
Σ σ ς	σίγμα	sīgmā	sig-mah	<b>s</b> in <b>sit</b> (unvoiced) <b>s</b> in <b>is</b> (voiced)	<b>s</b>
Τ τ	ταῦ	tau	tau	<b>t</b> in <b>talk</b>	<b>t</b>
Υ υ	ὕ ψιλόν	ūpsīlon	ew-pseeh-lawn	<b>u</b> in <b>lute</b> (long) <b>u</b> in <b>put</b> (short)	<b>y, u</b>
Φ φ	φῖ	phī	fee	<b>ph</b> in <b>phone</b>	<b>ph</b>
Χ χ	χῖ	chī	khey	<b>ch</b> in <b>chemist</b>	<b>ch</b>
Ψ ψ	ψῖ	psī	psee	<b>ps</b> in <b>psalm</b> (initial) <b>ps</b> in <b>lips</b> (medial)	<b>ps</b>
Ω ω	ὦ μέγα	ō mēgā	oh-may-gah	<b>o</b> in <b>note</b>	<b>ō</b>